Modeling and Evaluating Forecasting of Market Index Volatility: Evidence from Pakistani Stock Market

*Fauzia Mubarik **Dr. Attiya Y. Javid

* PhD Scholar, Shaheed Zulfiqar Ali Bhutto Institute of Science and Technology
** Pakistan Institute of Development Economics, Islamabad

Abstract

The present study investigates the evaluation and forecasting of volatility in the market index of Karachi stock exchange of Pakistan for the time period of July 1998 to June 2011. This investigation is demonstrated through the application of ARCH family of models identified as symmetric GARCH and GARCH-M and asymmetric EGARCH, GJR, TARCH and PGARCH models based on in-sample fit and out-sample fit forecasts. The error statistics of these forecasts are then measured under the error terms identified as root mean squared error, mean squared error through bias proportion, variance proportion and covariance proportion, Theil-inequality coefficient. These error statistics are used to check the accuracy of the models that best provide forecasts. Of all the asymmetric models, only EGARCH model has a negative leverage effect indicating that bad news decreases volatility and good news increases volatility. The in-sample forecasting performance of the symmetric and asymmetric GARCH models measured through three error distributions find that the student-t distribution shows more accuracy in forecasting behavior as compared to other distributions. In the out-of-sample forecasting performance check, the asymmetric models are better and they out-perform from that of the symmetric models. Conclusively, of all the error distributions, the student-t distribution shows efficiency in the predictability of stock volatility of market returns during the period 1998-2011 and that the asymmetric models are more appropriate in modeling the Pakistani market index volatility.

Keywords: Forecasting Volatility, Error Distributions, ARCH Family of Models.

Introduction

Volatility is the rate at which the price of the security moves up and down and is meant to be a key input in various investment decisions and portfolio creations. The volatility is high when the price of the security creates inertia by moving up and down rapidly over short intervals of time and is low when the price almost never changes.

Predictability of stock volatility is supposed to be important in investment decisions, security valuation techniques, risk management process, macroeconomic policy making. As predictability is always uncertain based on intuition and guess-work, it is not considered to be same as risk but is helpful in assessing investment risk. Regarding investors, the forecasting of stock volatility plays an integral role in the assessment of future returns depending upon which the investors plan to take an investment risk. Caiado (2004) consider volatility as a non-static variable following a time varying process. The author suggests that periods of high volatility tend to be more persistent as compared to low volatility. The dataset used by the author comprises of daily and weekly PSI-20 index of Lisbon and Oporto Stock Exchange (BLVP) for the time period of January 2, 1995 to November 23, 2001. The author concludes that EGARCH model provides better daily forecasts for multistep forecasting and GARCH model with index involved in variance equation provides better weekly forecasts.

With an increase in the common use of derivative securities by financial institutions, the phenomenon of forecasting stock volatility has played an integral role in the assessment of future security returns. Anderson, Bollerslev, Diebold & Labys (2003) have designed a framework of volatility forecasting based on value-at-risk calculation in asset pricing and allocation as well as risk management applications. The value-at-risk calculation comprises of two key characteristics; first, it exploits all information in intraday data and second, it is very easy to apply for high-dimensional return volatility modeling. The sample selected by the author comprises of thirteen years of spot quotations with currencies of U.S. dollars, Deutschemark and Japanese yen spot exchange rate markets over the time period of 1986 to 1999. Based on long memory Gaussian Vector Autoregression method, the result shows more successful volatility forecasts. These forecasts when tied with parametric lognormal-mixture of distribution, it generates standardized predictions of future returns and associated quantile forecasts.

This research study investigates the evaluation and forecasting of volatility in the market index of Karachi Stock Exchange of Pakistan for the time period of July 1998 to June 2011. This investigation is demonstrated through the application of ARCH family of models identified as symmetric GARCH and GARCH-M and asymmetric EGARCH, GJR, TARCH and PGARCH models based on in-sample fit and out-sample fit forecasts. The error statistics of these forecasts are then measured under the errors terms identified as root mean squared error (RMSE), mean squared error (MSE) and Theil-Inequality coefficient. These error statistics are used to check the

accuracy of the models that best provide forecasts through bias proportion, variance proportion and covariance proportion.
There is very scarce literature available regarding Pakistani stock market to demonstrate the predictability behavior of stock volatility through the application of ARCH family models. This study attempts to fill this gap by analyzing the evaluation and the forecast ability of stock volatility at the market index of Karachi stock exchange (KSE-100).

Rationale of the Study

The study attempts to model volatility of market index KSE-100 index using the market trading volume and the evaluation of forecasting performance. The mean equation comprising of previous volatility and previous residuals are considered as the explanatory variables. The variance equation takes six specifications: GARCH, GARCH-M, EGARCH, TGARCH, GJR and PGARCH. Secondly, three error distributions: normal distribution (Gaussian), student-t distribution and generalized error distribution (GED) are considered for in-depth modeling of volatility respectively. Thirdly, the evaluation of all models that best provides forecasts of stock volatility is measured through the error statistics identified as root mean squared error (RMSE), mean squared error (MSE) through bias proportion, variance proportion and covariance proportion and Theil-Inequality coefficient

Objectives of the Study

The main objective of the study is to demonstrate the forecasting of stock volatility in the market index of Karachi stock exchange of Pakistan for the time period of July 1998 to June 2011. This is examined through the application of ARCH family models both symmetric and asymmetric. The objectives are as follows:

- To model market return volatility using market volume by applying symmetric non-linear models: normal GARCH, GARCH-M and asymmetric: EGARCH, GJR, TGARCH and PGARCH.
- Estimation of all the six models with three assumptions regarding error distribution: normal distribution, student-t distribution and generalized error distribution.
- Generation of in-sample fit forecast and out-sample fit forecasts of market return volatility.
- Evaluation of the forecasting performance of the 18 models under three measures the root mean square error, the mean absolute error, Theil

inequality coefficient through bias proportion, variance proportion and covariance proportion.

Organization of the Study

The plan of the study is as follows. This study is further divided into four sections. Section two includes literature review, Section three explains the data and methodology, section four is comprised of data analysis and interpretation of empirical results and section five explains the summary and conclusion.

Literature Review

Gokbulut & Pekkaya (2014) examines the volatility forecasting of the Turkish stock market, the interest rate and the foreign exchange market (basket) by using the daily data for the time period of Jan. 2, 2002 to Feb. 4, 2014 respectively. Based on the various GARCH volatility models employed in the study, the authors find the TGARCH and the CGARCH models to be the most robust and the superior forecasting volatility models respectively.

In another study by Truck & Liang (2012) the authors have studied the forecasting volatility behavior of the gold market for the time period of Jan 4, 1999 to Dec 30, 2008 respectively. Based on the GARCH volatility models, the authors have found the TGARCH model to be the superior model to forecast volatility of the gold market respectively.

Ahmed & Suliman (2011) study the volatility forecasting behavior of the Khartoum stock exchange for the time period of Jan. 2, 2006 to Nov. 30, 2010 respectively. The authors have employed both symmetric and asymmetric GARCH volatility models and find significant results of the symmetry as well as the asymmetry in the respective stock market.

Kavocic (2008) has critically examined various GARCH models that accurately forecasts volatility. This ARCH family comprises of symmetric (GARCH) and asymmetric GARCH types of models identified as EGARCH, GJR, TARCH and PGARCH. The sample consists of high frequency closing market index MBI-10 from Macedonian stock exchange for the time period of Jan. 4, 2005 to Sep. 21, 2007. Based on normal, student–t and generalized error distribution (GED), authors conclude that GARCH models with non-Gaussian error distributions increase elasticity in the modeling of the conditional variance.

Chou (2005) has proposed the Conditional Autoregressive Range (CARR) model to forecast volatility in relation with standard deviation GARCH. This model is used as it creates ease in estimation by Quasi-

Maximum Likelihood Estimation method. It is applied on the daily index data with open, close, high and low of the Standard and Poor 500 for the time period of Apr. 26, 1982 to Oct., 17, 2003. To investigate the efficiency level between ECARR and GARCH, the author has performed 100 out-of-sample forecasts. The result shows that CARR produce efficient estimates of as compared to GARCH at range interval of every hour and every quarter.

McQueen & Vorkink (2004) have developed a preference-based equilibrium model that demonstrates the long-run stock predictability, excess volatility and volatility clustering. The data set comprises of monthly returns for the time period of 1802 to 2000, the daily returns for the time period of 1885 to 2000, 30-day Treasury bill for the time period of 1926 to 2000, and consumption wealth ratio for the time period of 1959 to 2000. Authors have used a mental scorecard based on prior investment performance that is helpful in assessing stock volatility. The authors conclude that GARCH asymmetry clearly gives evidence in the prediction of stock volatility and the scorecard is significantly correlated with future volatility as indicated by the preference-based equilibrium model.

Poon & Granger (2005) have studied four types of volatility-forecasting methods namely historical volatility, ARCH models, stochastic volatility and option-implied volatility. The sample consists of S&P 500 returns for the time period of February 1983 to July 2003. The authors have used four types of daily volatility proxies and conclude that the forecasting power for stock index volatility is 50-58 percent for horizons of 1 to 20 trading days. Out of all forecasting methods, the option-implied volatility dominates time series models because the market option price completely includes current information and future volatility expectations. Nevertheless, all the models equally cover one another's gaps in one way or the other.

Timmermann (1996) has examined three components that contribute to the volatility of stock prices under learning. These components are rational expectations, an estimation error term and covariance between rational expectations and estimation errors. The simulation results show that uncertainties about the parameters of dividend process leads towards the dynamic behavior of stock prices but are not suffice in explaining excess volatility in stock prices. The author suggests that predictability of stock prices needs a forecasting model rather a simple dividend equation and that the agents' learning significantly increase the volatility of stock prices.

Yu (2002) has demonstrated and compared the performance of nine alternative models for predicting stock price volatility in the New Zealand stock market. The data set comprises of 4741 daily returns of 40 largest and most liquid stocks for the time period of Jan. 1, 1980 to Dec. 31, 1998. The models are identified as the random walk, historical average, moving average, simple regression, exponential smoothing, exponentially-weighted moving average, ARCH and GARCH and the SV models. These models are tested through four evaluation measures identified as root mean square error (RMSE), the mean absolute error (MAE), the Theil-U statistic and the LINEX loss function. The authors conclude that according to RMSE, Theil-U and three asymmetric loss functions, the most appropriate model to forecast volatility is the SV model. They further suggest that in addition to the added information there is some contribution of the other variables such as inflation rates and the number of listed companies in improving the out-of-sample forecasting performance.

Brailford & Faff (1996) have examined the accuracy of various forecasting models in the Australian stock market in predicting the volatility in stock returns. The data set comprises of Statex-Actuaries Accumulation Index comprising of 50 most actively traded companies on the Australian stock exchange. The sample consists of more than 4900 observations for the time period of Jan. 1, 1974 to Jun. 30, 1993. The forecasting models are identified as random walk model, historical mean model, moving average models, exponential smoothing model, exponentially weighted average model, simple regression model, standard GARCH models and GJR-GARCH models. These models are tested through four measures identified as the mean error (ME), the mean absolute error (MAE), the root mean squared error (RMSE), and the mean absolute percentage error (MAPE). The authors suggest that all models' superiority on one another in forecasting volatility is based on the choice of error statistic. The authors conclude that a simple regression model and the ARCH class of models provide accuracy in forecasting volatility in stock returns as compared to other models. The authors have recognized GJR-GARCH model as the most appropriate model in forecasting volatility in the Australian stock returns.

Rashid & Ahmad (2008) have examined forecasting of stock index volatility by using linear and nonlinear methods. The data set comprises of the daily closing prices of KSE-100 index for the time period of January 2001 to November 2007. The linear methods employed by the authors are a random walk, an autoregressive model, a moving average model, an exponential smoothing model and a double exponential smoothing model. The non-linear methods used by them are generalized ARCH-in-M (GARCH-M) model, exponential GARCH (EGARCH) model and power

ARCH (PARCH) model. The authors conclude that the exponential smoothing model is the most appropriate linear model to forecast volatility based on root mean squared error. Amongst the non-linear models, they find that the GARCH-in-mean (1,1) model is the appropriate model to forecast volatility. The authors further conclude that the non-linear ARCH-class models out-perform the linear models in an out-of-sample stock price index forecasting volatility.

Hamilton & Lin (1996) have evaluated the forecasting of stock return volatility and economic turning points based on one key factor identified as economic recessions. The data set comprises of monthly stock return for the time period of 1965:1 to 1993:6. The authors have used bivariate models illustrated as model A, model B, model C, model C', model D and model D' to test for forecasting volatility in stock returns. The authors conclude that Model C shows superiority over other models providing better forecasts of stock volatility.

Magnus & Fosu (2006) have demonstrated on the forecasting of volatility of returns on the Ghanaian stock market with the application of GARCH models. The data set comprises of daily closing prices of the Ghana Stock Exchange Databank Stock Index (DSI) for the time period of Jun. 15, 1994 to Apr. 28 2004. The sample comprises of 1508 observations excluding the public holidays. The authors have used the basic random walk model, a symmetric GARCH (1,1) model and two asymmetric EGARCH (1,1) and TGARCH (1,1) models based on Akiake information criteria and the maximum log-likelihood values a set of model diagnostic tests. The author's conclude that the non-linear symmetric GARCH (1,1) model is the most appropriate model to forecast the conditional volatility of the stock index.

Goyal (2000) has demonstrated the implication of GARCH models in forecasting volatility in stock returns. The author has compared these models with ARMA model to check the robustness of each model in predicting volatility in stock returns. The data set comprises of the daily and monthly series of the CRSP value weighted returns including dividends for the time period of July 1962 to December 1998. The models are identified as GARCH, EGARCH, AGARCH and GJRGARCH. These models are applied on the in sample and out sample tests based on root mean squared error (RMSE) criteria. The author concludes that the volatility series obtained from GARCH models easily capture the entire variation in actual volatility. It is further suggested that a simple ARMA model outperforms than a GARCH-M model.

Anderson, Bollerslev, Diebold & Labys (2003) have demonstrated the value-at-risk calculation for volatility forecasting and conditional return fractile. The data set comprises of all interbank Deutschemark/U.S.dollar and the Japanese yen/U.S. dollar bid/ask quotes spot exchange rates for the time period of Dec. 1, 1986 to Jun. 30, 1999. The authors have developed framework from continuous-time arbitrage-free price processes that is run through long memory Gaussian vector autoregressive-realized volatility (VAR-RV) to forecast stock volatility. They conclude that the VAR-RV outperform in forecasting volatility than various GARCH models implemented by authors in studying volatility.

Pesaran & Timmermann (2000) have studied the modified version of recursive modeling strategy capable of forecasting stock volatility. The dataset comprises of the monthly returns on the FT All Share Index for the time period of 1965-1993. Based on the non-parametric test, the authors conclude that recursive model selection provide better forecasts and improve on the risk-return trade-off offered by the market portfolio. The authors suggest that this predictability could get inefficient for investors who do not use publicly available information because of their systematic over reaction to news.

Awartani (2008) investigates the noise effects on the forecasting performance of the heterogeneous autoregressive realized volatility model (HAR-RV). The dataset comprises of 30 Dow Jones Industrial Average stocks for the time period of Jan. 1, 1997 to Dec. 31, 2002. Based on the tri-power variation measures, the model is implemented on various sampling intervals. The authors conclude that by decomposing realized volatility on 15 minutes sampling interval, more accuracy is observed in the out-of-sample forecasts and starts declining when the interval is changed to 5-minute interval. They suggest that accuracy in forecasts is high when the noise effect is weak derived from the jump information on sampling intervals.

Marquering & Verbeek (2004) have examined trading strategies capable of exploiting the predictability in the stock returns and volatility. The mean-variance investor is studied to closely analyze the forecasting behavior. The data set comprises of S & P 500 index and the 3-month and 12-month treasury bills for the time period of January 1966 to August 2001. The simple linear models are employed and estimated recursively. The authors conclude that the predictability is higher when volatility is high. The authors further find that for a mean-variance investor, this predictability is appropriate no matter no short sales are allowed and the transaction costs are high.

Cooper, Downs & Patterson (2000) explains the relationship between the trading volume and real estate returns based on the asymmetric information in the speculative market. The authors use the filter-rule methodology as it easily detects the non-linearities in the predictability of price changes and Wang model for research analysis. The sample comprises of the weekly returns and weekly volume for 301 Real Estate Investment Trusts (REITs) from Wednesday-close to Wednesday-close over a period of 1973 to 1995. Those securities are included that have a daily volume in the previous 10 trading days. Under the filter-rule methodology, six portfolio strategies are developed. The first-two strategies are the price-only strategies and are named as the loser-price and winner-price strategies. The remaining four contains the price and volume information and are named as loser-price-low volume, loser-price-high volume, winner-price-low volume and winner-price-high volume. It is found that consistency regarding price-volume dynamics remains throughout the periods and the predictability of returns varies between high and low volume periods. It is also found that the behavior of returns is affected by private information and heterogeneous investment opportunities.

Franses, Leif & Paap (2002) have demonstrated a model useful in modeling and forecasting level shifts in absolute returns of nine stock markets. The model is termed as Censored Latent Effects Autoregressive (CLEAR) model capable of capturing the long-memory properties in the data. The dataset comprises of Dow Jones (USA), the NASDAQ (USA), the S&P 500 (USA), the Nikkei-225 (Japan), the FTSE-100 (Great Britain, the DAX Xetra (Germany), the CAC-40 (France), the AEX (the Netherlands), and the Hang Seng (Hong Kong) for the time period of 1990-1999. The authors have included a technical trading rule type explanatory variable having a predictive power for future level shifts. The CLEAR model is further extended to long memory switching regime CLEAR (SR-CLEAR) model to capture the leverage effects. These models are compared with ARFIMA and ARFIMA-X models to check the accuracy and appropriability of either model in forecasting absolute returns. The authors conclude that the CLEAR models out-perform in forecasting the future shift levels and future stock returns.

Methodology and Data

This is an attempt to investigate the forecasting performance of market return volatility including the market volume in the models. Firstly, this study analyzes the market return volatility by applying symmetric models: normal GARCH and GARCH-M and also asymmetric models:

EGARCH, TGARCH, GJR, and PGARCH. Thereafter, three assumptions are made regarding the error term distribution for more in-depth analysis for all these six models: normal distribution, student-t distribution and generalized error distribution. In the next stage, in-sample and out-of-sample forecasting is done for all 18 models. Finally, the evaluation of forecasting is performed by mean square error, mean absolute error, Theil inequality coefficient. The market index and market turnover is collected from the website of Business Recorder for the period of July 1998 to June 2011 comprising of 3450 observations from the website of Business Recorder.

Modeling the Return Volatility

In evaluating the forecasting performance of volatility of stock returns and market returns the ARCH family of models is used. The analysis begins with symmetric models: normal GARCH model and GARCH-M in which volume is used in the mean and variance equation. Then, to capture asymmetry EGARCH, GJR and Power ARCH is used.

GARCH Model

The analysis begins with return-volume relation in mean equation by regressing the market return on past market return and past volume. The variance equation includes past error square, past volatility and also past volume. Therefore, the model is described as follows:

Mean equation $r_{it} = \phi_0 + \phi_1 r_{it-1} + \phi_2 V_{it-1} + \varepsilon_{it}$ (1)

Variance equation $\sigma_i^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$ (2)

GARCH-in-mean model. The GARCH-in-mean model has been proposed by Engle, Lillian, & Robins (1987) and the return volume with GARCH (1,1)-M model is expressed as;

Mean equation $r_{it} = \phi_0 + \phi_1 r_{it-1} + \phi_2 V_{it-1} + \lambda\sigma_i + \varepsilon_{it}$ (3)

Variance equation $\sigma_i^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$ (4)

Asymmetric GARCH Models

Empirical evidence suggests that volatility tends to accelerate when the stock market index is decreasing. This behavior of volatility that the positive shocks affect more than the negative shock is called asymmetric volatility could be explained by leverage effect and time varying risk premium called the volatility feedback. This study estimates this asymmetric relationship in volatility by following GARCH models;

Exponential GARCH model. The exponential GARCH (EGARCH) model has been proposed by Nelson (1991) that explains the leverage effect by incorporating volume in the mean and variance equation given by:

Mean equation $r_{it} = \phi_0 + \phi_1 r_{it-1} + \phi_2 V_{it-1} + \lambda\sigma_i + \varepsilon_{it}$ (5)

Variance equation

$$\log\sigma_i^2 = \omega + \alpha\left[\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - E\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)\right] + \beta\log\sigma_{t-1}^2 + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (6)$$

GJR Model. The GJR model proposed by Glosten, Jagannathan & Runkle (1993) captures the difference between the effect of negative and positive news. The model is expressed as;

Mean equation $r_{it} = \phi_0 + \phi_1 r_{it-1} + \phi_2 V_{it-1} + \varepsilon_{it}$ (7)

Variance equation $\sigma_i^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma I\varepsilon_{t-1}^2$ (8)

Threshold GARCH model. The threshold GARCH (TGARCH) is another asymmetric variant of GARCH model proposed by Zakoïan (1994) that models the conditional standard deviation instead of the conditional variance as shown below;

Mean equation $r_{it} = \phi_0 + \phi_1 r_{it-1} + \phi_2 V_{it-1} + \varepsilon_{it}$ (9)

Variance equation $\sigma_i = \omega + \alpha\varepsilon_{t-1} + \beta\sigma_{t-1} + \gamma I\varepsilon_{t-1}$ (10)

Power GARCH model. Power GARCH (PGARCH) model is another asymmetric model introduced by Ding, Granger and Engle (1993) explaining the impact of positive and negative news on the asset prices expressed as;

Mean equation $r_{it} = \phi_0 + \phi_1 r_{it-1} + \phi_2 V_{it-1} + \varepsilon_{it}$ (11)

Variance equation $\sigma_i^\delta = \omega + \alpha(|\varepsilon_{t-1}| + \gamma\varepsilon_{t-1})^\delta + \beta\sigma_{t-1}^\delta$ (12)

Alternative Distribution Conditions

For more in-depth analysis of modeling volatility this study has specified all GARCH-type models mentioned above are based upon three assumptions about the error distribution: Normal Gaussian distribution, Student−*t* distribution and Generalized Error Distribution (GED). The explanation is that it is more appropriate to assume that the errors have a heavy tailed distribution rather than normal distribution. Student- *t* distribution is expressed as:

$$f\langle z_t \mid \nu\rangle = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)\Gamma\left(\frac{\nu}{2}\right)}\left[1+\frac{z_t^2}{\nu-2}\right]^{\frac{\nu+1}{2}}}$$

Where $z = \varepsilon_t / \sigma_t$, $\Gamma(.)$ is gamma function and v>2 is the shape parameter.

Generalized error distribution is proposed by Nelson (1991) and it is expressed as:

$$f\langle z_t \left| v \right\rangle = \frac{v}{\lambda_v . 2^{(v+1)/v}\Gamma(1/v)} \exp\left[-\frac{1}{2}\left\|\frac{z_t}{\lambda_v}\right\|^v\right], \lambda_v = \left[\frac{2^{(-2/v)}\Gamma(1/v)}{\Gamma(3/v)}\right]^{1/2}$$

Where Γ(1/υ) is a gamma function and υ is the positive shape parameter.

Forecasting Evaluation

After modeling the forecasting behavior of market return (KSE 100), the next step is to evaluate the forecasting performance of GARCH models. To test how well these different models fit the past data in-sample forecasting is generated. In addition for further evaluation of forecasting performance out-of-sample forecasting is also done.

The in-sample fit and the out-sample fit forecasting performance of each model is analyzed by using three symmetric measures; the root mean square error (RMSE*)*, the mean absolute error (*MAE)* and the Theil inequality coefficient (*TIC)* respectively. These measures are expressed as follows;

$$RMSE = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2}$$

Where $\hat{\sigma}_t^2$ denotes volatility forecast, σ_t^2 denotes volatility in actual

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{\sigma}_t^2 - \sigma_t^2\right|$$

The Theil inequality coefficient is defined by;

$$TIC = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2 - \sigma_t^2)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t^2)^2}\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\sigma_t^2)^2}}$$

The Theil inequality coefficient value lies between zero and zero indicates a perfect fit.

Empirical Results

Data and Sample

The data set comprises of market index of Karachi Stock Exchange for the time period of July 1998 to June 2011. The trading turnover of market of KSE 100 is obtained. The first step is to calculate market return by taking the log first difference of KSE-100 index closing prices. The market volume is constructed as natural logarithm of market turnover. The market index and market turnover are tested for stationarity by applying Augmented Dicky Fuller test and are turned stationary by taking log first difference. The volume series is detrended to make it stationary.

Regression Results

The data have been implemented on the symmetric and asymmetric GARCH models to check their feasibility and accuracy in predicting the future returns. The evaluation of forecasting is performed by mean square error, mean absolute error and Theil inequality coefficient. The tables below show the in-sample and out-of-sample forecasts by the ARCH family models.

The analysis begins by estimating the symmetric models: normal GARCH and GARCH-M and asymmetric model with assumption of normal error distribution. In the mean equation market return is regressed on lag market return and lag market volume. Both of these explanatory variables are significant at 1% level as shown in the results reported in Table 1. As regards the GARCH mean term λ, it is significant only in the GARCH-M and PGARCH model indicating that investor are getting negative compensation of volatility risk. As far as variance equations, only EGARCH shows asymmetry behavior indicating that negative shocks affect volatility more than positive shocks. In all six models both ARCH term and GARCH terms are significant mostly at 1% level.

Table 1. *Estimates of Volatility Models with Normal Distribution*

	GARCH	GARCH-M	EGARCH	GJR	TARCH	PGARCH
			Mean Equation			
ϕ_0	0.001* (3.87)	-0.006 (-3.04)	-0.008* (-14.85)	-0.06* (-2.55)	-0.006 (-2.58)	-0.06* (-5.32)
ϕ_1	0.07** (3.14)	0.07* (3.07)	0.07* (3.62)	0.07* (3.31)	0.07* (3.31)	-0.08 (3.74)
ϕ_2	0.03** (1.67)	0.04** (3.83)	0.001* (9.95)	0.004* (3.40)	0.004* (3.39)	0.04* (5.81)
λ		-0.04** (-0.49)	-0.14 (-2.52)	-0.06 (-0.95)	-0.06 (-1.94)	-0.11** (-1.82)

			Variance Equation			
ω	0.001*	0.00*	-0.90	0.01*	0.001	0.005**
	(16.1)	(13.4)	(-21.4)	(14.7)	(14.76)	(1.89)
α	0.17*	0.18*	0.34*	0.18*	0.06	0.19*
	(15.6)	(15.9)	(3.39)	(15.4)	(13.15)	(16.50)
β	0.78*	0.78*	0.92*	0.77*	0.77*	0.14*
	(9.80)	(9.86)	(19.8)	(10.00)	(10.04)	(3.55)
γ			-0.03*	0.09*	0.06*	0.79*
			(-3.39)	(3.20)	(3.15)	(7.74)
δ						0.17*
						(9.03)
R^2	0.51	0.54	0.58	0.57	0.58	0.68

The results reported in Table 2 shows the estimates of all six models with the assumption of error distribution follows student t distribution. In these models, the volatility risk λ has premium in four models: GARCH-M, EGARCH, TGARCH, and GJR. The variance equation of EGARCH shows asymmetric behavior. All other results remain the same as in normal distribution.

Table 2. *Estimates of Volatility Models using Market Return with Student^{-t} Distribution*

	GARCH	GARCH-M	EGARCH	GJR	TARCH	PGARCH
			Mean Equation			
ϕ_0	-0.07	-0.03	-0.004*	-0.02	0.004	-0.04
	(-3.15)	(-0.13)	(-6.92)	(-0.07)	(-0.17)	(-1.15)
ϕ_1	0.07**	0.08*	0.06*	0.09*	0.09*	0.07*
	(3.19)	(4.06)	(3.22)	(4.10)	(4.10)	(5.23)
ϕ_2	0.04**	0.02	0.04*	0.09***	0.003*	0.02*
	(3.72)	(1.80)	(6.88)	(1.59)	(1.88)	(5.45)
λ		0.08*	-0.09*	0.08*	0.07*	-0.07
		(3.67)	(-2.36)	(3.77)	(3.70)	(-0.50)
			Variance Equation			
ω	0.006*	0.001	-0.74	0.03	0.03	0.05*
	(4.54)	(4.41)	(-8.24)	(0.76)	(0.74)	(3.11)
α	0.29*	0.29*	0.44*	0.56*	0.50*	0.25*
	(8.23)	(8.29)	(12.93)	(9.19)	(8.46)	(15.9)
β	0.75*	0.75*	0.95*	0.70*	0.71*	0.58*
	(3.71)	(3.75)	(10.59)	(14.74)	(5.20)	(3.54)
γ		0.30*	-0.04**	0.02	0.04	0.18*
		(3.45)	(-1.72)	(0.55)	(0.63)	(3.26)
δ						0.45
						(9.33)
R^2	0.49	0.53	0.57	0.56	0.57	0.64

The Table 3 shows the results of all six models with error distribution assumed to be GED. Most of the results are same as in other distributions. The GARCH-M term λ is significant in EGARCH, GJR and TGARCH models. All models except EGARCH model show no asymmetry in volatility.

Table 3. *Estimates of Volatility Models using Market Return with GED Distribution*

	GARCH	GARCH-M	EGARCH	GJR	TARCH	PGARCH
			Mean Equation			
ϕ_0	-0.003* (-3.45)	-0.03 (-1.34)	0.004* (-5.73)	-0.004 (-1.26)	-0.002 (-1.26)	-0.06 (-1.33)
ϕ_1	0.05** (4.60)	0.06* (3.08)	0.06* (3.07)	0.06* (3.04)	0.06* (3.04)	0.05* (2.72)
ϕ_2	0.03** (3.30)	0.04* (2.03)	0.004* (5.91)	0.02* (2.11)	0.002* (2.10)	0.05* (3.06)
λ		-0.03 (-0.59)	-0.07*** (1.69)	-0.05** (-1.81)	-0.05** (-1.74)	0.12 (1.33)
			Variance Equation			
ω	0.001* (7.07)	0.001 (6.90)	-0.81 (-9.91)	0.001* (6.92)	0.05 (6.91)	0.001 (0.61)
α	0.23* (8.64)	0.24* (8.80)	0.43* (11.38)	0.24* (8.60)	0.20* (6.43)	0.13 (1.28)
β	0.76* (4.19)	0.76* (4.28)	0.92* (9.53)	0.75* (4.04)	0.75 (4.04)	0.59* (4.29)
γ		0.55* (3.66)	-0.04* (-1.79)	0.10* (2.14)	0.09* (2.08)	0.05 (0.09)
δ						0.20 (3.03)
R^2	0.50	0.54	0.57	0.57	0.58	0.66

Interpretation of Results

Tables 1, 2 and 3 show the estimates of the volatility models using market return at three distribution levels of Gaussian, Student-t and GED. These models comprises of the mean equation and the variance equation. The mean equation includes two past return values capturing the direct process in the return series and the autocorrelation coefficients of the market returns following an autoregressive process of order 2. The coefficient λ describes the relationship between returns and their volatility and is significant at 5% or 10% level. The variance equation includes three coefficients: ω (consant), α (ARCH effect) and β (GARCH effect). The estimated parameters α and β in the GARCH type models determine the short-run dynamics of the volatility. The coefficient λ (leverage effect) describes the return-volatility relationship and is significant at 5% or 10% level. In most asymmetric models, the leverage effect is significant at the 5% or 10% level based on non-normal distribution. Of all the asymmetric models, only EGARCH model has a negative leverage effect indicating that volatility is decreased due to bad news and simultaneously volatility is increased due to good news respectively. The findings indicate that to model the Pakistani market index volatility the asymmetric models are the suitable models to estimate volatility.

Table 4. *In- Sample Forecasting Performance of Volatility Models GARCH (Normal)*

	Root Mean Sq. Error	Mean Ab. Error	Mean Ab. %age Error	Theil Inequ. Coeff.	Bias Prop.	Var. Prop.	COV. Prop.
GARCH (Normal)	0.0476	0.0136	196.66	0.965	0.0004	0.0004	0.015
EGARCH (Normal)	0.0481	0.0149	649.06	0.864	0.0032	0.7009	0.296
TGARCH (Normal)	2.2E+1	1.7E+1	2.9E+1	1	0.006	0.9940	2.1E-11
GJR (Normal)	2.1E+1	1.7E+1	5.8E+1	1	0.006	0.9937	2.2E-154
PGARCH (Normal)	0.0476	0.0138	144.11	0.9769	0.0005	0.9553	0.044

Table 5. *In- Sample Forecasting Performance of Volatility Models GARCH (student^{-t})*

	Root Mean Sq. Error	Mean Ab. Error	Mean Ab. %age Error	Theil Inequ. Coeff.	Bias Prop.	Var. Prop.	COV. Prop.
GARCH (student^{-t})	0.0476	0.0136	222.28	0.958	0.0008	0.980	0.019
EGARCH (student^{-t})	0.0476	0.0136	189.53	0.969	0.0002	0.979	0.021
TGARCH (student^{-t})	2.0E+41	2.8E+40	1.8E+45	1	0.0193	0.981	4.4E-43
GJR (student^{-t})	1.9E+41	2.6E+40	1.6E+45	1	0.0193	0.981	4.8E-43
PGARCH (student^{-t})	0.0476	0.0136	170.20	0.974	0.0001	0.989	0.011

Table 6. *In- Sample Forecasting Performance of Volatility Models GARCH(GED)*

	Root Mean Sq. Error	Mean Ab. Error	Mean Ab. %age Error	Theil Inequ. Coeff.	Bias Prop.	Var. Prop.	COV. Prop.
GARCH (GED)	0.048	0.014	213.92	0.960	0.0006	0.985	0.0145
EGARCH (GED)	0.048	0.014	201.75	0.964	0.0005	0.986	0.0133
TGARCH (GED)	2.8E+47	3.6E+46	1.9E+51	1	0.0167	0.983	3.3E-49
GJR (GED)	0.048	0.014	211.55	0.963	0.0004	0.971	0.0287
PGARCH (GED)	0.048	0.014	143.86	0.977	0.0006	0.955	0.0441

Interpretation of Results

The tables 4, 5 and 6 show the in-sample forecasting performance of the symmetric and asymmetric GARCH models. Based on three error distributions, the student-t distribution shows more accuracy in forecasting behavior as compared to other distributions. Likewise, normal GARCH and EGARCH models have shown very weak forecasting performance. The models that have out-performed are identified as GJR and PGARCH models in the in-sample forecasting.

Table 7. *Out of Sample Forecasting Performance of Volatility Models GARCH (Normal)*

	Root Mean Sq. Error	Mean Ab. Error	Mean Ab. %age Error	Theil Inequ. Coeff.	Bias Prop.	Var. Prop.	COV. Prop.
GARCH (Normal)	0.05	0.014	198.36	0.966	0.0002	0.98	0.022
EGARCH (Normal)	0.02	0.020	1202.64	0.701	0.0010	0.06	0.94
TGARCH (Normal)	4.3E+8	4.6E+7	1.5E+8	1	0.0114	0.99	7.6E-83
GJR (Normal)	2.2E+7	2.4E+7	7.8E+7	1	0.0124	0.99	1.5E-74
PGARCH (Normal)	0.05	0.014	160.96	0.973	0.0003	0.95	0.05

Table 8. *Out of Sample Forecasting Performance of Volatility Models GARCH (student^t)*

	Root Mean Sq. Error	Mean Ab. Error	Mean Ab. %age Error	Theil Inequ. Coeff.	Bias Prop.	Var. Prop.	COV. Prop.
GARCH (student^{-t})	0.021	0.014	300.60	0.871	0.059	0.60	0.338
EGARCH (student^{-t})	0.021	0.014	304.78	0.869	0.055	0.61	0.333
TGARCH (student^{-t})	0.048	0.014	170.12	0.972	0.0001	0.94	0.055
GJR (student^{-t})	0.048	0.014	170.04	0.972	0.0001	0.94	0.055
PGARCH (student^{-t})	2.1E-02	0.015	341.18	0.854	0.048	0.58	0.370

Table 9. *Out of Sample Forecasting Performance of Volatility Models GARCH(GED)*

	Root Mean Sq. Error	Mean Ab. Error	Mean Ab. %age Error	Theil Inequ. Coeff.	Bias Prop.	Var. Prop.	COV. Prop.
GARCH (GED)	0.047	0.014	239.67	0.957	0.0005	0.949	0.051
EGARCH (GED)	0.021	0.014	336.82	0.856	0.0475	0.592	0.360
TGARCH (GED)	0.021	0.014	285.75	0.876	0.0670	0.593	0.340
GJR (GED)	0.048	0.014	300.25	0.941	0.0016	0.938	0.060
PGARCH (GED)	0.020	0.015	419.60	0.819	0.0197	0.633	0.348

Interpretation of Results

Tables 7, 8 and 9 show the out-sample fit forecasting performance of symmetric and asymmetric GARCH models. The performances of asymmetric models are better and they out-perform from that of the symmetric models. Of all the error distributions, the student^{-t} distribution shows efficiency in the predictability of stock volatility of market return during the period 1998-2011.

Summary and Conclusion

This study investigates the evaluation and forecasting of volatility in the market index of Karachi stock exchange of Pakistan for the time period of July 1998 to June 2011 comprising of 3450 observations from the website of Business Recorder. This investigation is demonstrated through the application of ARCH family of models identified as symmetric GARCH and GARCH-M and asymmetric EGARCH, GJR, TARCH and PGARCH models based on in-sample fit and out-sample fit forecasts. The error mean statistics of these forecasts are then measured under the errors terms identified as root mean squared error (RMSE), mean squared error (MSE), Theil-inequality coefficient through bias proportion, variance proportion and covariance proportion. These error statistics are used to check the accuracy of the models that best provide forecasts. Of all the asymmetric models, only EGARCH model has a negative leverage effect indicating that bad news would decrease volatility and good news would increase volatility. The findings show that the asymmetric models are more appropriate in modeling the Pakistan market index volatility during 1998 to 2008. The in-sample forecasting performance of the symmetric and asymmetric GARCH models measured through three error distributions find that student-^{t} distribution shows more accuracy in forecasting behavior as compared to other distributions. Likewise, normal GARCH and EGARCH models have shown very weak forecasting performance. The models that have out-performed are identified as GJR and PARCH models in the in-sample forecasting. In the out-of-sample forecasting performance, the asymmetric models are better and they out-perform from that of the symmetric models. Of all the error distributions, the student t- distribution shows efficiency in the predictability of stock volatility of market return during the period 1998-2008 and 2009-2011 respectively.

Policy Implications

The study helps in investigating the superior behavior of one of the generalized ARCH-M models to assess and evaluate the in-sample fit and out-sample fit forecasting performance of the market index volatility that could be useful for investors and macro-economic policy makers for portfolio construction, security valuation and risk management.

References

Ahmed, A. E. M., & Suliman,S. Z. (2011). Modeling stock market volatility using garch models evidence from sudan. *International Journal of Business and Social Science, 2(*23*),* 114-128.

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003). Modeling and forecasting realized volatility. *Econometrica*, *71*(2), 579-625.

Awartani, B. (2008). Forecasting volatility with noisy jumps: an application to the Dow Jones Industrial Average stocks. *Journal of Forecasting*, *27*(3), 267-278.

Brailsford, T. J., & Faff, R. W. (1996). An evaluation of volatility forecasting techniques. *Journal of banking & Finance, 20*(3), 419-438.

Caiado, J. (2004). Modelling and forecasting the volatility of the portuguese stock index PSI-20. *Estudos de Gestão, 9*(1), 3-22.

Chou, R. Y. T. (2005). Forecasting financial volatilities with extreme values: the conditional autoregressive range (CARR) model. *Journal of Money, Credit, and Banking*, *37*(3), 561-582.

Cooper, M., Downs, D. H., & Patterson, G. A. (2000). Asymmetric information and the predictability of real estate returns. *The Journal of Real Estate Finance and Economics*, *20*(2), 225-244.

Franses, P. H., Van Der Leij, M., & Paap, R. (2002). Modelling and forecasting level shifts in absolute returns. *Journal of Applied Econometrics*, *17*(5), 601-616.

Gokbulut, R. I., & Pekkaya, M. (2014). Estimating and forecasting volatility of financial markets using asymmetric garch models: an application on turkish financial markets. *International Journal of Economics and Finance, 6*(4), 23-35.

Goyal, A. (2000). Predictability of stock return volatility from GARCH models. *Anderson Graduate School of Management, UCLA, Los Angeles, CA, May.*

Hamilton, J. D., & Lin, G. (1996). Stock market volatility and the business cycle. *Journal of Applied Econometrics*, 11(5), 573-593.

Kovacic, Z. (2008). Forecasting volatility: Evidence from the Macedonian stock exchange. *International Journal of Finance and Economics,* Issue: 18

Magnus, F. J., & Fosu, O. (2006). Modeling and forecasting volatility of on returns on the Ghana stock exchange using GARCH models. *Journal of Applied Science, 3*(10) 2042-2048.

McQueen, G., & Vorkink, K. (2004). Whence GARCH? A preference-based explanation for conditional volatility. *Review of Financial Studies*, *17*(4), 915-949.

Marquering, W., & Verbeek, M. (2004). The economic value of predicting stock index returns and volatility. *Journal of Financial and Quantitative Analysis*, *39*(2).

Pesaran, M. H., & Timmerman, A. (2000). A recursive modeling approach to predicting UK stock returns. *The Economic Journal*, *110*(460), 159-191.

Poon, S. H., & Granger, C. (2005). Practical issues in forecasting volatility. *Financial Analysts Journal*, *61*(1), 45-56.

Rashid, A., & Ahmad, S. (2008). Predicting stock returns volatility: An evaluation of linear vs. nonlinear methods. *International Research journal of Finance and Economics, 20,* 141-150.

Timmermann, A. (1996). Excess volatility and predictability of stock prices in autoregressive dividend models with learning. *The Review of Economic Studies*, *63*(4), 523-557

Truck, S., & Liang, K. (2012). Modelling and forecasting volatility in the gold market. *International Journal of Banking and Finance*, *9*(1), 48-80.

Yu, J. (2002). Forecasting volatility in the New Zealand stock market. *Applied Financial Economics*, *12*(3), 193-202.

Yalama, A., & Sevil, G. (2008). Forecasting world stock markets volatility.
International Research Journal of Finance and Economics, *15,* 159-174.