NUML International Journal of Business & Management
Volume 21, Issue 1, June (2026)
Journal Home Page: nijbm.numl.edu.pk/index.php/BM
ISSN 2410-5392 (Print), ISSN 2521-473X (Online)

Asymmetric Multifractal Detrended Cross-Correlation Patterns in the Dynamics of the Green Investment Funds and Financial Markets: Insights from geopolitical risk

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Article History:
Received: April 7, 2026
Revised: May 23, 2026
Accepted: June 10, 2026
Available Online: June 30, 2026

Keywords: Geopolitical risk; Green investment funds; Sustainable finance; Asymmetric multifractal detrended cross-correlation analysis

ABSTRACT

The rapid expansion of sustainable finance has increased the importance of green investments for global investors, especially during geopolitical uncertainty and financial market instability. This study investigates the nonlinear and asymmetric dependence structure among geopolitical risk, green investment funds, and financial indices. It also examines the multifractal behavior of their cross-correlations. For this purpose, daily data from 14th October 2013 to 18th September 2025 is analyzed using Asymmetric Multifractal Detrended Cross-Correlation Analysis. This method enables the study to assess whether geopolitical risk affects green and conventional markets differently across varying scales of fluctuation. The results show nonlinear dependence and long-range cross-correlations between geopolitical risk and financial markets. Strong multifractal characteristics are observed across all asset pairs, although the degree if persistence differs across markets. The findings suggest that green investment funds are more vulnerable to geopolitical risk than some conventional financial assets. The Hurst exponent results reveal stronger persistence during periods of small fluctuations, whereas larger fluctuations are associated with weaker persistence and greater instability. Evidence from the multifractal spectrum also points to asymmetric market behavior, implying that positive and negative shocks affect markets differently. These results have important implications for both investors and policymakers in improving risk management and supporting the long-term resilience of sustainable finance.

1. Introduction

Climate change, energy transition and sustainable development have gained new prominence in the global financial system, as investors, firms and policymakers are increasingly confronted with them. Today, green investment funds, including clean energy stocks, green bonds, and socially responsible investments (SRIs), are an important source of funding for environmentally sustainable projects and for driving a low-carbon economy.

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Apart from their environmental significance, they are also utilized for portfolio diversification and risk management. Hence, the growing scope of sustainable finance has made it imperative to analyze the behavior of green investment funds under increased uncertainty.

Meanwhile, green investment markets are nascent and can be susceptible to external shocks. Geopolitical risk (GPR) has been gaining attention as a major source of uncertainty in financial markets. According to Caldara and Iacoviello (2022), GPR is the threat of war, terrorism, nuclear proliferation, and military escalations that threaten peaceful international relations. In the past few decades, geopolitical tensions have been impacted by the Brexit Referendum, the Chinese stock crash, the US-China trade war, the COVID-19 pandemic, the Russian-Ukrainian war, and the Israel-Palestine war. These events have been instrumental in shaping outcomes in global financial and energy markets (Arouri et al., 2025), potentially affecting investor expectations, risk aversion, liquidity, and asset price dynamics.

The effect of geopolitical risk on financial markets can occur through different mechanisms. Increased geopolitical tensions can dampen confidence, suppress consumption and investment, disrupt capital flows, and contribute to volatility in financial and commodity markets. Under these circumstances, it is common for investors to adjust their portfolios, reallocating funds away from riskier investments and towards safer and/or alternative assets. This could trigger geopolitical impacts on stock markets, energy markets, metals, cryptocurrencies, and safe-haven assets, including gold. A growing body of research has investigated these effects in stock markets (Aslam et al., 2015; Yang et al., 2021; Sharif et al., 2020), cryptocurrencies (Colon et al., 2021; Kyriazis, 2021), metals (Baur & Smales, 2020; Yilanci and Kilci, 2021), energy (Alsagr and Van, 2021; Liu et al., 2019), and oil markets (Silsu et al., 2021; Liu et al., 2019; Su et al., 2019).

However, the link between geopolitical risk and green investment funds has not been studied extensively. This matter is worth consideration, as green investment funds are not autonomous but are linked to broader financial and energy markets. Fossil fuels are well-entrenched in the global energy system, and renewable energy, electric vehicles, and investment in climate change continue to grow at a fast pace. There is a close relationship between crude oil and macroeconomic conditions, inflationary pressure, geopolitical uncertainty, and energy security. Likewise, gold remains a safe investment when times are uncertain. Therefore, oil and gold can be important vectors by which geopolitical events could affect the rest of the financial system.

Green financial instruments have become part of the wider financial system. Their performance is now tied more tightly to capital markets, energy markets, investor sentiment and climate policy uncertainty. Recent studies indicate that the relationship between green bonds and other sustainable financial instruments and commodity and energy markets can be complex, nonlinear, and scale-dependent (Acikgoz et al., 2024; Asl et al., 2025). This indicates that green investment funds may also be exposed to global market turbulence. Given this, it is important to investigate whether geopolitical uncertainty affects green investment funds the same way as it affects traditional financial securities, or whether it affects them differently depending on the size and scope of fluctuations.

Traditional theories such as the Efficient Market Hypothesis (EMH) are limited in explaining these market dynamics. The EMH principle assumes that asset prices are fully determined by the information available to investors and that they tend to fluctuate randomly. However, financial time series are frequently characterized in practice by long memory, volatility clustering, fat tails and non-linear dependence. These features have made the Fractal Market Hypothesis (FMH) more relevant to financial markets, which are systems with multifractal structures and scale-dependent behavior. In such dynamics, multifractal methods can be effective in characterizing patterns of persistence, complexity and dependence that are not necessarily captured by standard linear models. Fractal theory has also been used extensively to explain the complexity of financial markets. Fractals have been useful for studying the irregular, yet structured patterns found in economic and financial time series since Mandelbrot's early work. There are two major methods used: monofractal and multifractal analysis. Monofractal models can be useful for detecting persistence and long memory, but may be too restrictive to apply to financial markets, where dependence varies in scale, and fluctuations are highly heterogeneous. Multifractality, however, offers a more meaningful approach to the complexity of financial time series than mono-fractality (Mandelbrot, 1997).

Although research on geopolitical risk and financial markets has developed, several gaps remain. First, most studies are limited to traditional financial markets, oil, metals, cryptocurrencies or energy markets and the relationship between geopolitical risk and green investment funds has been less studied. Second, although financial markets react to uncertainty non-linearly and asymmetrically, most studies that use existing data methods are linear and symmetric. Third, there is little evidence on how green investment funds adjust their responses to geopolitical risk relative to traditional indices, bonds, commodities, and alternative assets. Lastly, the persistence and multifractal characteristics of the cross-correlations between geopolitical risk and green financial assets remain poorly understood, particularly across different fluctuation scales.

To fill these gaps, this paper investigates cross-correlation between geopolitical risk, green investment funds, and traditional financial market indexes, with a focus on non-linear, asymmetric, and multifractal relationships. The study is based on daily data from 14 October 2013 to 18 September 2025 and uses Asymmetric Multifractal Detrended Cross-Correlation Analysis to measure differences in the effects of geopolitical risk on green and conventional financial markets across different levels of volatility. The method is suitable as it can detect nonlinear dependence, asymmetric behavior, long-range cross-correlations, and market responses to changes in scale, which are not usually measurable by traditional linear models.

The primary objective of this research is to investigate the multifractal and asymmetric dependence structure between geopolitical risk and green investment funds, as compared to traditional financial assets and other alternative assets. This study aims to achieve four main objectives. First, it analyzes whether there are notable cross-correlations between geopolitical risk and green investment funds, and whether these correlations are multifractal. Second, it studies the cross-correlation dependence on small and large fluctuation scales. Third, it assesses if there is an asymmetric response to geopolitical risk by green investment funds in varying market conditions. Fourth, it tests whether the behavior of the green investment funds differs from that of conventional indices, bonds, commodities, gold and Bitcoin in times of geopolitical uncertainty.

This study contributes to understanding how geopolitical uncertainty is transmitted to sustainable financial markets. Despite the general perception of green investment funds as instruments of carbon transition and sustainable development, their financial behavior in the face of geopolitical risk has not been sufficiently explored. The volatility of geopolitical risk and its relationship with green assets are a concern for investors, as they can affect portfolio diversification, hedging, and sustainable investment strategies. It also applies to policymakers because geopolitical shocks can undermine investor confidence in sustainable finance markets and disrupt the flow of green capital. This study explores nonlinear and scale-dependent dependence structures to provide evidence that could guide investors, portfolio managers, and regulators in better understanding the vulnerability and resilience of green financial assets amid global uncertainty.

This study contributes to the literature in four ways. First, it extends the geopolitical risk (GPR) literature by focusing on green investment funds alongside conventional financial markets. Previous research focused on traditional financial markets or on energy assets, but this study investigates the multifractal cross-correlation between green investments and geopolitical risk. This brings new insights into the role of sustainable finance in addressing global geopolitical uncertainty and extends the use of multifractal approaches to green assets. Secondly, the study elucidates the dynamic relationship between geopolitical risk and green investments, as it is able to depict how the assets react and respond to various scales of fluctuation. This can help to explain the complex nonlinear relationship between geopolitical risk and green investment funds, which has been under-researched in previous studies. Third, the use of MF-ADCCA enables the study to measure the asymmetric multifractal cross-correlations and understand the internal complexity and local characteristics of green investments as a function of geopolitical risk. This method is particularly useful for identifying subtle cross-correlation patterns over different time scales, which has important implications for risk diversification and investment strategies. The study provides valuable evidence for portfolio managers and policy makers interested in the effects of geopolitical risks on sustainable financial markets by analyzing the multifractality of geopolitical risk and green investment. Fourth, the study offers practical implications as it demonstrates the possible impact of geopolitical risks on green financial assets at varying fluctuation levels.

Overall, the results offer a better understanding of the effects of geopolitical uncertainty on both green and traditional financial markets. Investors and portfolio managers may find the results useful in their diversification, hedging, and risk-management strategies. For policymakers, the findings may help in assessing the vulnerability of sustainable finance markets during geopolitical crises. The research fills the broader sustainable finance literature in two ways: first, by highlighting the need to consider green investment funds not just as environmental drivers, but also as transmitters of risk, as dependent on markets and as actors in periods of global uncertainty.

2. Literature Review

The growing importance of green investment funds and financial markets reflects the wider shift toward sustainable investment practices. The world economy is constantly changing and Environmental, Social and Governance (ESG) criteria have become an integral component of contemporary portfolio management. This trend is especially apparent in green investment funds, which primarily invest in a wide range of green activities, including renewable energy, green bonds, and low carbon projects. These markets are projected to grow significantly, with Naeem et al. (2023) predicting ESG assets would grow to $53 trillion by 2025. Several studies have examined the behavior and efficiency of green financial assets. For instance, Fernandes et al. (2023) examined the weak form of the Efficient Market Hypothesis (EMH) along with multifractality of green bonds, stock indexes and US bonds using Multifractal Detrended Cross-Correlation Analysis (MF-DCCA). Their results show that green bonds are characterized by nonlinear correlations and multifractal behavior. Likewise, Zhuang and Wei (2022) used asymmetric multifractal detrended fluctuation analysis to evaluate the efficiency of green finance markets, traditional stock indices, and oil prices. Their findings reveal strong multifractality in both upward and downward movements of green finance indices, suggesting that these markets remain inefficient under both small and large fluctuations. Green finance and, especially, green bonds have gained much attention within the past 20 years from both academics and investors. This interest can be attributed primarily to two reasons. First, the global transition to the sustainable and low-carbon economy has raised the importance of green bonds as a financing tool for projects relating to climate and environment purposes. Flaherty et al. (2020) highlights the role of green bonds in addressing environmental challenges, while Flammer (2020) explains how green bonds can support corporate environmental sustainability. Second, previous empirical evidence suggests that green bonds have relatively low correlations with traditional financial assets, which makes them useful for diversification and hedging purposes. Guo and Zhou (2020) demonstrate that green bonds can be used as effective hedge items, especially in stock and foreign exchange markets during the volatile period. Additionally, Jiang et al. (2021) validate that green bonds can help hedge in the medium term.

The popularity of green bonds among investors and financial practitioners has been increasing; one of the reasons for this is that they can help diversify portfolios during periods of instability in financial markets. They also have value in diversifying a portfolio, particularly in times of economic crisis. Reboredo and Ugolini (2021), Reboredo (2021), and Tang and Zhang (2020) study the risk transmission in green bond markets and find that there are significant diversification benefits in different financial markets. This means that green bonds not only appeal to investors who value environmental awareness but also to traditional investors focused on risk management and improving portfolio returns.

However, recent research has increasingly indicated that green financial assets are not insulated from financial-market dynamics, especially in uncertain times. The study by Arfaoui et al. (2025) investigates the potential of green investment funds to mitigate climate risk and highlights the increasing importance of green funds as sustainability and risk-management instruments. This is similar to the present study as the geopolitical risk is an external uncertainty shock that can impact the dependence structure among green investment funds and conventional financial markets. Furthermore, the link between clean energy, green markets and cryptocurrencies suggests that green assets are increasingly linked to the traditional and non-traditional financial markets. On the basis of this literature, this research examines the effects of geopolitical risk on green investment funds and major financial markets using an asymmetric multifractal detrended cross-correlation approach in various market situations and time horizons.

Although green markets have grown rapidly, the effect of geopolitical risk on green investment markets remains relatively underexplored. Political instability, wars, energy disruptions, and oil price shocks can lead to significant volatility in global financial markets and have a significant impact on asset prices in various sectors. It is crucial for investors and policymakers to be aware of how these external shocks impact green investment funds in order to build more effective investment strategies and promote sustainable long-term growth. One advanced method for examining the complex and asymmetric relationships between green investment funds and traditional financial markets is Asymmetric Multifractal Detrended Cross-Correlation Analysis (AMF-DCCA). This strategy allows for the detection of the multifractal characteristics, market inefficiencies and cross-market dependence among various fluctuation levels including during times of stress like geopolitical crises. AMF-DCCA is more comprehensive than the linear models as it considers the asymmetry, nonlinearity and scale-dependent dynamics of the market. Arfaoui et al. (2023) study the dependence relationship between green markets, clean energy, and cryptocurrencies in a related study. The results indicate that these markets have experienced greater dependence during the pandemic, while green bonds do not seem to be as dependent and may potentially provide diversification benefits. This allows for the integration of both green and traditional financial assets in the current research: Bitcoin, ESG assets, renewable energy assets, and main market indices. They also found that under normal circumstances, the average correlation between green finance and cryptocurrency assets is low, while under stress circumstances, the correlation between them is high. This is another reason why the use of AMF-DCCA is more appropriate than linear correlation only models.

2.1 Geopolitical risk and green markets

Geopolitical risk is now important for energy security, financial stability and environmental sustainability. Caldara and Iacoviello (2022) define geopolitical risk as the risk associated with terrorism, interstate tensions, and war. Previous research indicates that geopolitical risk can affect international trade, political relations, investment activities, and security. Geopolitical tensions can influence financial market expectations of future returns, delay market adjustment and pose threats to critical market infrastructure (Bouoiyour et al., 2019; Wang et al., 2024; Ren et al., 2024). The risks associated with the main energy exporting and energy importing countries have also increased, posing threats to the financial markets' stability around the world (Liu et al., 2021; Chan et al., 2024). Structural supply shocks are now known to rapidly seep into both energy and financial markets, given recent geopolitical events. For example, the war between Russia and Ukraine in 2022 put strong upward pressure on energy prices, underscoring the close link between geopolitical uncertainty and energy market instability. This is particularly important for green and clean energy and socially responsible markets, as they play a pivotal role in the global energy transition and are affected by strategic uncertainty and market volatility. Helmi et al. (2024) examined the effect of geopolitical risk on these markets and find that GPR has a significant impact on sustainable assets, with varying effects depending on the time period and trading strategies. In particular, whereas GPR has a negative effect in the short run, it has a positive effect in the long run. The short-term negative effect is due to elevated uncertainty, resource scarcity, and risk aversion associated with geopolitical tensions (Sivaram and Saha, 2018; Sweidan, 2021). In contrast, the long-term positive impact may arise from the way geopolitical tensions foster clean energy development, renewable energy uptake, and strategic investment in green energy technologies (Dutta and Dutta, 2022). In this regard, Su et al. (2021) suggest that geopolitical factors can reshape clean energy development by promoting institutional and policy reform. ESG assets could also gain greater market legitimacy and access to funding, helping them to withstand geopolitical risks (Fiorillo et al., 2024).

The link between geopolitical risk (GPR) and renewable energy investment has gained prominence in the financial markets, energy security, and sustainable development literature. GPR impacts energy stocks, financial markets, and renewable energy investment through its effects on investor sentiment, risk assessment, and investment, which can lead to greater volatility in energy assets (Sarker et al., 2023). Qin et al. (2020) show evidence that the impacts of GPR on returns and volatility of different energy sources are not uniform, while Zhao et al. (2023) indicate that renewable energy can serve as a dynamic hedge against GPR. These insights suggest that GPR has a complex impact on renewable energy markets. It may reduce investment by raising uncertainty, but it may also stimulate investment by increasing the importance of energy security and green energy generation strategies. This is also enhanced by the environmental and social impacts of fossil fuel-based energy systems, such as climate change, energy access inequality, and energy poverty. In this regard, renewable energy investment is a key factor in sustainable development, contributing to SDG 7 on clean energy, SDG 13 on climate change, and SDGs 14 and 15 on life below water and on land, respectively (Liu et al., 2023). However, the existing literature still lacks sufficient evidence on the interaction between geopolitical risk and climate policy uncertainty and its effects on renewable energy investment. Yang et al. (2021) further note that geopolitical risk in energy-producing nations can cause energy supply disruptions, influence crude oil prices, and increase market uncertainty. The Russia-Ukraine war of 2022 serves as a key example, significantly affecting energy markets and creating new incentives for renewable energy investment (IEA, 2022a). The conflict highlighted the fragility of energy security and the importance of transitioning to renewable energy sources (Iyke, 2024).

Geopolitical risk (GPR) is multifaceted and can affect renewable energy investments. Rising geopolitical uncertainty can, on the other hand, raise project costs and deter private-sector investment in renewable energy markets. However, it can also spur the government to expedite investment in renewable energy to enhance long-term energy security and lessen reliance on external energy sources (Bashir et al., 2023). Furthermore, renewable energy assets are frequently seen as a good diversifier against geopolitical and economic policy volatility, adding to their appeal as investment vehicles for mitigating risk exposure (Zhao et al., 2023). But the empirical evidence isn't conclusive. Some studies suggest that geopolitical risk positively influences renewable energy investments, whereas Javed et al. (2025) identify a negative relationship, indicating that the sign and strength of this relationship is still not clear. Although the literature on renewable energy markets is expanding, there is greater emphasis on consumption patterns, price dynamics, and environmental outcomes than on investment behavior. Most studies examine renewable energy consumption (Li et al., 2023; Ullah et al., 2023; Xi et al., 2023), clean energy price volatility and returns (Sarker et al., 2023), and environmental quality (Assamoi and Wang, 2023), while largely overlooking renewable energy investment itself. Moreover, many of these studies rely on proxy indicators such as energy production or consumption, which may not accurately capture financial investment decisions or real market responses in the renewable energy sector. A further limitation is the lack of research quantifying renewable energy investment in monetary terms, which is essential for understanding real capital flows into clean energy projects. A geopolitical shock also has significant consequences for the overall energy system. Any disruption to global supply chains can lead to instability in energy imports and uncertainty in crude oil markets. In response, economies may temporarily increase their use of coal and other fossil fuels, resulting in higher carbon emissions and upwards pressure on the price of emissions trading system (ETS). The growing geopolitical risk increases the volatility of fossil fuel supply chains which can lead to broader economic impacts, especially if energy flows are suddenly curtailed (Zhao et al., 2023). The Russia-Ukraine conflict, for example, dramatically highlighted the fragility of fossil fuel markets, showing the risks associated with overdependence on conventional energy sources (Meckling et al., 2022). According to Tang et al. (2026), GPR increases reliance on fossil fuels, enhancing the release of CO_2.

As geopolitical uncertainty persists, there is a clear incentive for countries to pursue energy diversification, with increasing investments in renewable energy being a strategic move to reduce fossil fuel dependency and reduce risks related to energy security (Alsagr and Van Hemmen, 2021; Meckling et al., 2022). Given this backdrop, green markets have been offering carbon-efficient equities as an attractive investment option. These equities are viewed as promising due to their potential to deliver stable long-term returns in a world transitioning toward sustainable, low-carbon growth. Carbon-efficient companies are increasingly seen as less exposed to the risks of climate change and regulatory policies, which make them safer and more profitable investments for those concerned with long-term environmental impact.

The global community faced substantial disruptions in early 2020 with the simultaneous onset of the COVID-19 pandemic and the Russian invasion of Ukraine, introducing unprecedented challenges to global economies. The conflict between Russia and Ukraine has fundamentally changed the global agenda and its energy markets, as well as international supply chains. Such changes have prompted governments to rethink their policies and introduce new strategies to tackle the growing domestic economic pressures (Banerjee et al., 2024). Meanwhile, the ongoing conflict in the Middle East is escalating and has raised the levels of geopolitical tensions, another example of how those risks are becoming more difficult to manage in a more interdependent global economy. As geopolitical uncertainty becomes more persistent, governments are assuming a more active role in maintaining economic stability and supporting the transition towards resilient economic systems. Such developments are also visible in financial markets, as uncertainty has brought up an elevated risk premium, and to some extent led to higher volatility in traditional financial instruments and green investment funds.

During geopolitical turbulence, financial markets often display simple linear behavior. For asset returns, such as stocks, bonds and crude oil, Shi and Guo (2025) contend that the dynamics are complex, nonlinear and multi-scale in the presence of geopolitical risk. Specifically, market behavior differs according to small and large markets fluctuations, suggesting that market responses are different across time horizons and market conditions. The effect of geopolitical risk (GPR) on stock market volatility has a few interrelated channels. An increase in geopolitical uncertainty can cause economic decision-making processes to be postponed, which can translate into family members delaying their consumption decisions, and firms delaying investment and expansion plans (Bloom, 2009; Salisu et al., 2022). Second, the escalation of geopolitical tensions reduces international trade and cross-border capital investment, which pushes companies to move away from global production networks and into more regionalized operations, thereby driving up production and operating expenses (Eckstein & Tsiddon, 2004). Third, geopolitical uncertainty raises the risk associated with financial investments. When the economy slows down and the market becomes less secure, investors often shift money from the riskier assets to other investment choices. This flight-to-safety effect exacerbates the fluctuations of stock prices and volatility of the stock market. These theoretical arguments are supported by empirical evidence. For instance, Fiorillo et al. (2024) investigates the connection between geopolitical uncertainty and the risk of stock market crashes and find that geopolitical uncertainties, especially those linked to external conflicts, greatly raise the likelihood of stock market crashes. Their findings suggest that heightened geopolitical uncertainty reinforces investor fear and risk aversion, leading to more severe and frequent market downturns. The findings are in line with the existing literature suggesting that geopolitical risk has a negative impact on investors' confidence and on financial markets volatility. Crude oil is one of the most important commodities in the global economy with significant implications for governments, businesses and financial investors (Pal et al., 2014).

Crude oil is the primary energy source and fluctuations in its prices impact broadly on macroeconomic outcomes. The relatively inelastic demand for oil also means that a rise in the price of oil is likely to be passed through to higher production costs and inflationary pressures across the economy. Therefore, it is important to study the dynamics of crude oil prices to understand how energy price shocks ripple through financial markets. In times of greater uncertainty, gold plays a similar important role. It is considered to be a safe-haven asset and is a popular hedging tool against inflation, deflation, and currency devaluation. Although there is a vast body of literature on oil and gold markets, little empirical study exists on the cross-correlation between oil/gold markets and green investment funds, especially during inflationary periods in response to geopolitical shocks. Addressing this gap is important for understanding how geopolitical risk is transmitted across traditional commodities and sustainable financial assets

2.2 Theoretical Framework and Hypotheses Development

While there is useful evidence on geopolitical risk, green finance, energy markets, and conventional financial assets in them, there are some gaps. Previous studies have focused on the effect of geopolitical risk on energy prices, green investment, stock market volatility, and green energy markets individually. For instance, Caldara and Iacoviello (2022) offer a widely accepted measure of geopolitical risk, while studies like Qin et al. (2020), Yang et al. (2021), Zhao et al. (2023), Sarker et al. (2023), Helmi et al. (2024), and Javed et al. (2025) explore the impact of geopolitical risk on energy markets, renewable energy investment, and sustainable financial markets. Likewise, Fernandes et al. (2023) and Zhuang and Wei (2022) demonstrate the existence of multifractal features in green bonds, green finance indices, traditional stock markets, and oil prices. But the studies are either focused on green finance efficiency, renewable energy investment, energy markets, or specific sustainable indices, rather than the joint cross-correlation between geopolitical risk, green investment funds, and broader financial market indices in a comprehensive fashion. Another significant gap is in terms of methodology. Many existing studies have used traditional econometric models, volatility, and time-frequency techniques that have been valuable but could not account for the effects of nonlinearity, asymmetry, and scale. Although MF-DCCA and other multifractal methods have been used to study green bonds, energy markets, and financial assets, little has been said about the asymmetric multifractal cross-correlations between geopolitical risk and portfolios of green and conventional financial assets. This is relevant because financial markets may be sensitive to small and large fluctuations, positive and negative market movements. Thus, the literature leaves a gap regarding whether green investment products are more exposed to geopolitical risk than other assets when considering the effects of nonlinearity, long memory, and asymmetry.

To address this, the present study contributes to existing literature in several respects by using the AMFDCCA approach to investigate the asymmetric multifractal cross-correlation between geopolitical risk and green investment funds, as well as the major financial assets of Bitcoin, gold, bonds, global stock indices, and oil. This study examines whether the dependence between geopolitical risk and selected assets varies across fluctuation sizes, market conditions, and investment categories. Unlike studies that focus mainly on linear correlations or the effect of geopolitical risk on a single asset class, this research captures asymmetric, nonlinear, and scale-dependent cross-correlations across green and conventional financial markets. This gives us insight into the transmission of geopolitical risk across green and conventional investment assets. The literature reviewed leads to the following hypotheses:

H1: *Green investment assets exhibit asymmetric and scale-dependent multifractal dependence with geopolitical risk compared with traditional financial assets.*
H2: *The asymmetric cross-correlation between geopolitical risk and asset markets differs between upward and downward GPR trends.*
H3: *Geopolitical risk is associated with nonlinear and multifractal cross-correlations across green investment assets and traditional financial markets.*

The financial determinants underlying the market efficiency of green investment funds and financial markets can be examined using theoretical frameworks such as the efficient market hypothesis and stakeholder theory.

2.2.1 Efficient market hypothesis

Market efficiency suggests that asset prices incorporate available historical data at present (Cuestas et al., 2017). As new information emerges in the marketplace, prices are likely to respond rapidly to these new signals which move towards their intrinsic values (Gil-Alana et al., 2018). The Efficient Market Hypothesis (EMH) is a key concept in financial economics, first introduced by Fama (1970). Under the EMH, a financial market is efficient if all relevant information is incorporated into the price of assets. In the context of market efficiency, it is difficult to forecast asset prices due to their random fluctuations. As a result, there is no clarity in the systematic patterns in prices, which implies that investors cannot earn abnormal profits on a consistent, risk-adjusted basis through arbitrage.

Assessing the market efficiency of green investment funds is therefore important for investors, as it can provide insights into forecasting ability, arbitrage limitations, and portfolio risk-management strategies. Numerous findings related to long-range dependence, autocorrelation, and price reversals in asset returns (Poterba et al., 1988; Fama & French, 1988). The EMH typically assumes that the dynamics of markets are linear and independent. However, Liu et al. (2025) have contended that the EMH theory is based on the premise of homogenous information sets, while market participants are heterogeneous and operate under uncertainty. The difference underscores the need for alternative ways to deal with market complexity. Crises that are not only geopolitical but also health-related, such as the Russian invasion of Ukraine and the COVID-19 pandemic, have elevated cross-market spillovers of tail risk and co-movement across markets, potentially culminating in systematic crises. Market efficiency is defined by Fama (1970) in three forms: weak, semi-strong, and strong. The weak form of market efficiency suggests that the current stock price incorporates all past market information such as stock prices and volumes. The semi-strong form adds to this by suggesting that stock prices reflect all publicly available information, such as corporate earnings, stock splits, and macroeconomic data. The strong form takes it a step further by stating that stock prices reflect all information, both public and private, suggesting that insiders cannot outperform the market.

The efficient market hypothesis (EMH) has been extensively used to investigate emerging and developing markets, where increased volatility and return expectations drive market participants (Aktan et al., 2018; Khan and Khan, 2016). Many studies have investigated EMH based on stock price movements and price efficiency. In a random walk model, stock prices follow an unpredictable pattern, suggesting that past prices do not provide reliable information for predicting returns. Alternatively, if stock prices do not exhibit a random walk, there may be some systematic patterns that investors can exploit to forecast future returns based on historical prices. Under the weak form of EMH, all past price-related information is incorporated into the prices (Fama, 1970). While numerous studies have examined EMH, the results are not consistent. While some have found evidence of market inefficiency, others have reported evidence of market efficiency (Narayan and Smyth, 2004; Ozdemir, 2008; Marashdeh and Shrestha, 2008; Borges, 2010; Gozbasi et al., 2014; Rizvi et al., 2014; Tiwari and Kyophilavong, 2014; Neaime, 2015; Shahzad et al., 2016; Ali et al., 2018).

EMH is relevant for understanding the efficiency of both green investments and financial markets, and it serves as a foundation to examine whether green investments adhere to market efficiency principles under different geopolitical conditions. According to EMH, prices incorporate all available information, which limits investors' ability to gain excess returns from publicly available data. However, the multifractal and asymmetric behavior of green investment funds during geopolitical risk presents a unique opportunity to test the validity of EMH in the context of sustainable finance. By using the Asymmetric Multifractal Detrended Fluctuation Analysis (AMFDFA) and Asymmetric Multifractal Detrended Cross-Correlation Analysis (AMFDCCA), this paper examines whether green investments have non-random patterns and asymmetric responses, which would indicate inefficiencies, especially in situations of market stress and geopolitical instability. When green assets exhibit consistent correlations or non-linear price changes, this would be indicative of market inefficiencies, which would question the assumption of EMH. The research is thus useful in broadening the scope of market efficiency to include non-traditional assets, new evidence about the dynamic nature of green investments in different geopolitical environments, and practical solutions to investors and policymakers facing the challenges of sustainable finance.

The efficient green asset markets may offer the best financing sources to the riskiest investment project in case of green assets. On the other hand, ineffective financial markets will mislead green financing flows through diverting financial resources to less worthy green projects and neglecting projects that have good environmental connotations. It has been empirically proven that efficient markets can be a major contributor to green growth and structural change (Hu et al., 2021; Lee et al., 2023). By doing so, EMH gives a theoretical justification of optimal allocation of resources in green finance, which may result in sustainable economic development. Nevertheless, in case of market inefficiencies caused by mispricing of risk or inaccurate valuation of assets, this may bend the direction of green financing. Therefore, inefficient markets can focus capital on less worthy projects without considering those with good environmental implications. Such inefficiencies may result in misallocation of resources, which will result in suboptimal consequences of green investments and the sustainable development agenda in general. The empirical studies in green finance (Adekoya et al., 2021; Mensi et al., 2021) highlight that market inefficiencies impede the flow of capital to essential projects. Therefore, identifying the dynamics of market efficiency is crucial for policymakers aiming to establish mechanisms that facilitate fair asset pricing and direct capital toward projects with positive environmental impacts (Lyu et al., 2024; Wang & Wang, 2021).

2.2.2 Fractal Market Hypothesis (FMH)

The Fractal Market Hypothesis (FMH) proposed by Peters (1994) contradicts the classical EMH, stating that financial markets can be inefficient, non-linear and complex. Multifractality is a common characteristic of financial time series and is linked to market inefficiency and potential arbitrage opportunities, especially in emerging and ESG-focused markets. When such inefficiencies exist, investors may be able to generate abnormal returns. The distribution of price returns is not normally distributed under the FMH framework. Rather, they tend to exhibit leptokurtosis, fat tails, long-range dependence and nonlinear responses to new information (Zou & Zhang, 2020). The properties of FMH make it more capable of describing heavy-tailed distributions and big price swings than the linear assumption of EMH (Liu et al., 2019; Liu et al., 2020). Moreover, market efficiency can vary between bull and bear markets, and the level of asymmetry can suggest heterogeneity in risk responses and structural maturity of green finance markets (Zhuang & Wei, 2022).

The concept of long memory and multifractality is of particular interest for the analysis of price dynamics, as it may affect investment decisions, portfolio allocation, and policy design. According to the definitions proposed by Fama (1965, 1970), the presence of long memory challenges the core assumptions of the EMH. Long memory leads to predictability and regularity in the price and volatility of assets, which is in contradiction with the idea that the price movements are just random movements (Mandelbrot, 1997; Wang et al., 2010). The empirical evidence of long memory and multifractal patterns in green asset markets (i.e., green bonds, green equities and clean energy investments) suggests that these markets may not be fully efficient. These patterns suggest that the price movements may be recurring and can potentially identify arbitrage opportunities.

Such inefficiencies are especially notable in the green finance industry, where market prices tend to be more complicated than the random walk EMH. Further, FMH points out that markets are fractal; self-similarity is evident in various time scales. This fractality can be applied to the green asset markets and identify both the short-term and long-term market behavior patterns that cannot be explained by EMH (Liu et al., 2019b). Finally, FMH provides an improved interpretation of market complexity, including in green finance, where the market is usually effortlessly and non-predictably behaving, displaying the multifractal character of these markets and allowing more wizened investment approaches to be implemented.

2.3 Research Gap and Model

The preceding literature and theoretical discussion indicate that geopolitical risk has important implications for stock markets, energy markets, oil, metals, cryptocurrencies, and other conventional financial assets. However, the relationship between geopolitical risk and green investment funds remains relatively underexplored. In particular, limited evidence exists on whether green investment funds respond to geopolitical uncertainty differently from conventional assets across varying fluctuation scales and market conditions. This gap is important because green investment funds are closely linked with capital markets, energy-transition dynamics, climate-policy uncertainty, investor sentiment, and liquidity conditions. It means that their reaction to geopolitical risk will not necessarily be the same as that of traditional financial assets.

Another limitation of the existing literature is its reliance on linear, symmetric, and average-based models. These techniques give valuable information but fail to precisely reflect the non-linear dynamics, asymmetries, long-range cross-correlations, and scale-dependency of financial markets. To overcome these drawbacks, this study uses MF-ADCCA, a framework that is well suited for such analysis. The model is implemented in a pairwise setting between geopolitical risk and each selected green or conventional asset return series. This enables the study to determine whether the dependence structure is persistent, multifractal, asymmetric, and different across small and large fluctuations. In this way, the proposed framework provides a clear methodological bridge between the research gap identified in the literature and the empirical analysis presented in the following section.

3. Data and Methodology

This study investigates the dynamics between green investment funds and financial market indices, using a dataset that includes respective indices and funds with reliable historical data. The sample includes daily time-series data from selected green investment indices (Carbon Efficient index, MSCI socially responsible index, World Renewable Energy index, FTSE 4 Global Good index, S&P Global clean energy index, and S&P Green bond index) and major financial and cryptocurrency assets (S&P Global Developed Aggregate Ex-Collateralized Bond Index, S&P GSCI Crude oil, S&P GSCI Gold, MSCI ACWI index, and Bitcoin) along with the Geopolitical risk index (GPR).

The sample is selected based on a) data continuity and availability, b) market representation, c) market liquidity, and d) comparability. The study includes funds and indices with at least 12 years of observations to ensure consistency and reliability in multifractal analysis. To achieve the objectives of this study, secondary quantitative data is used. The data is obtained from the LSEG database, the MSCI official website, and the official S&P Global website. These platforms are recognized for their accuracy, comprehensive coverage, and consistency in providing financial and market data.
Table 1 shows the variables' descriptions used in our methodologies.

Table 1: Variables' Descriptions

Variable	Symbol	Definition	Source
Green Investment Funds			
1. Carbon Efficient Index	CARBON EFFICIENT	The Carbon Efficient Index is widely used as a benchmark for low-carbon equity investments because it reflects the performance of firms with lower carbon emissions while still maintaining exposure to the broader stock market.	LSEG
2. MSCI Socially Responsible Index (MSCI SRI)	MSCI_SRI	The MSCI socially responsible index is widely used to represent socially responsible stocks, as it includes companies with strong ESG characteristics while excluding companies engaged in controversial business activities.	LSEG
3. World Renewable Energy Index	RENEWABLE	The World Renewable Energy Index is commonly used as a benchmark for global renewable-energy equities because it captures listed firms operating in renewable and clean-energy activities across international markets.	LSEG
4. FTSE 4 Global Good Index	FTSE GOOD	The FTSE4Good Global Index is considered one of the most important ESG benchmarks and is widely used to represent global firms meeting defined environmental, social, and governance standards.	LSEG
5. S&P Global Clean Energy Index	CLEAN ENERGY	The S&P Global Clean Energy Index is a standard benchmark for global clean-energy equities and is commonly used to analyze the performance of firms involved in clean energy production and related services.	LSEG
6. S&P Green Bond Index	GREEN BOND	A global green bond benchmark representing bonds issued to finance environmentally sustainable projects	LSEG
Financial Markets			
MSCI All Countries World Index	ACWI	A global equity-market benchmark representing large- and mid-cap stocks across developed and emerging markets.	MSCI website
S&P Global Developed Aggregate Ex-Collateralized Bond Index	SPBOND	A global bond-market benchmark representing investment-grade government and corporate bond returns.	S&P website
S&P GSCI Crude Oil	CRUDE OIL	A crude-oil market benchmark representing global oil-market dynamics and conventional energy-market performance.	S&P website

S&P GSCI Gold	GOLD	A gold-market benchmark representing gold price performance and safe-haven commodity behavior.	S&P website
Cryptocurrency			
Bitcoin	BITCOIN	A cryptocurrency market proxy representing the most liquid and widely traded digital asset.	LSEG
Geopolitical Risk Index (GPR)	GPRD	An index measuring geopolitical uncertainty related to wars, terrorism, military tensions, conflicts, and international instability.	Caldara and Iacoviello (2022)

3.2 Study period

The study investigates a range of major geopolitical and economic crises between 2013 and 2025. The period 2013–2025 was selected to ensure a sufficiently long daily dataset for identifying multifractal and asymmetric cross-correlation patterns. AMFDCCA requires stable and turbulent market phases to capture small and large fluctuations, long-range dependence, and scale dependence of market reactions. The period encompasses stable market conditions and major market shocks, including the COVID-19 pandemic, energy price volatility, and geopolitical tensions. This diversification enhances the study's robustness by allowing us to examine how green investments and financial assets respond to geopolitical risk across diverse market environments. Each crisis serves as an exogenous shock, providing a substantial influence on global capital flows, asset valuations, and volatility spillovers. The analysis treats each event as a period of heightened uncertainty to assess changes in cross-market linkages.

The study employs several key variables derived from daily time-series data of both green investment funds and conventional financial market indices. The primary variable for analysis is the logarithmic return, which measures the continuously compounded rate of return for each asset. The logarithmic return for each series is computed as follows:

$$r_t = \ln\left(\frac{P_t}{P_t - 1}\right)$$

Here $r_{i,t}$ signifies the log return at time period t and $P_t,\ P_{t-1}$ represents the current and lagged prices, respectively. The use of a logarithmic transformation normalizes the data, mitigates scale effects, and captures proportional changes in asset values, making the series suitable for multifractal and efficiency analyses. The asymmetric multifractal detrended cross-correlation fluctuation analysis (AMFDCCA) method, proposed by Cao et al. (2014), is applied to return series to measure asymmetric multifractal cross-correlation, scaling behavior, and market efficiency under both upward and downward market trends. To capture the effect of geopolitical uncertainty, the study incorporates geopolitical risk (GPR) events following the framework of Caldara and Iacoviello (2022). Prior studies have used GPR index and event data to explore risk transmission and market efficiency (Bouri et al., 2018; Naeem et al., 2022; Zhang et al., 2023). GPR events are used here to represent volatility shocks arising from geopolitical conflicts such as wars, terrorist attacks, or economic disruptions.

3.3 Methodology

The MF-ADCCA is based on A-DFA and the MF-DCCA, and it allows for the examination of asymmetric multifractal cross-correlation characteristics among green investment funds, financial market indices, and geopolitical risk. MF-ADCCA is an extension of MF-DCCA that combines asymmetric DFA and is designed to capture cross-correlation under upward and downward trend conditions separately, rather than treating the full sample symmetrically. This is appropriate for financial systems because market responses often differ across positive and negative regimes, especially during periods of geopolitical tension.

A growing body of research shows that financial markets often display multifractal behavior. Multifractal modeling is valuable as it can account for some empirical properties of financial returns, such as volatility clustering, heavy-tailed distributions, scale-dependent behavior, and long memory (Baruník et al., 2012). However, older multifractal models face practical limitations due to their combinatorial complexity and non-stationarity, arising from the finite time intervals used. Earlier methods, such as rescaled range analysis and detrended fluctuation analysis, were primarily used to study monofractals in time series (Hurst, 1951; Peng et al., 1994). While rescaled range analysis was used in initial explorations of long memory in time series, it may be biased by short-range dependence. Detrended fluctuation analysis resolved some of these issues by enabling researchers to detect long-range dependence in non-stationary time series while minimizing the misleading effects of trends and non-stationarity (Green et al., 2014).

More recently, Kantelhardt et al. (2002) further refined this method with multifractal detrended fluctuation analysis, allowing for a closer look at multifractal scaling in financial time series. It has been applied in many studies of crude oil, stock markets, and cryptocurrency markets (He et al., 2010; Aslam et al., 2021; Mnif et al., 2020). Following the principles of detrended fluctuation analysis, detrended cross-correlation analysis (DCCA) was developed to study long-range dependence between two non-stationary time series (Podobnik et al., 2011). But the scaling exponents obtained from DCCA can be unreliable, leading to overestimation or underestimation of multifractality (Ferreira et al., 2016). To address this problem, Zhou (2008) developed MF-DCCA, which is a combination of DCCA and MF-DFA, to measure multifractal cross-correlations between two financial variables. This approach offers a more reliable method for investigating the degree, complexity, and local structures of multifractal correlation between financial time series.

Consider two time series, $x_i$and y_i, where $i = 1,2, \ldots, N$, and N indicate the sample size. The MF-ADCCA approach is implemented through the following steps.

Step 1: Profile construction

First, both series are transformed into cumulative profiles after removing their mean values.

$$X(i) = \sum_{t=1}^{i} (x_t - \bar{x}), Y(i) = \sum_{t=1}^{i} (y_t - \bar{y}), i = 1, \ldots, N$$

where $\bar{x}$and $\bar{y}$ represent the average values of the two time series over the entire study period.

Step 2: Segmentation of the profiles

Next, the profiles $X(i)$and $Y(i)$are then into $N_s \equiv [N/s]$ consecutive segments of equal size s. When the total number of observations N is not perfectly divisible by the scale s, so, the last observations at the end may not be used. In order to use these observations, the profiles are segmented again from the other end, producing a total of $2N_s$segments.

Step 3: Local detrending and covariance estimation

In each of the $2N_s$segments, linear regression is applied to estimate the local trend of both profiles. The local trend functions are specified as follows:

$$X_v(i) = a_{X_v} + b_{X_v} i, Y_v(i) = a_{Y_v} + b_{Y_v} i$$

The detrended covariance is then computed after subtracting the estimated local trends from the original profiles. For the first $N_s$segments, where $v = 1, \dots, N_s$, the covariance follows the forward segmentation process. For the remaining segments, where $v = N_s + 1, \dots, 2N_s$, a backward segmentation order is used.

$$F(v,s) = \frac{1}{s}\sum_{i=1}^{s} \mid X[N-(v-N_s)s+i] - X^v(i) \mid . \mid Y[N-(v-N_s)s+i] - \mathrm{Y}^{\mathrm{v}}(i) \mid$$

Step 4: Construction of fluctuation functions

Next, the q-order fluctuation functions are obtained by averaging across all segments and separately considering upward and downward local trends in x_t. For $q \neq 0$. The upward and downward fluctuation functions are defined as follows:

$$F_q^+(s) = \left(\frac{1}{M^+} \sum_{v=1}^{2N_s} \frac{\mathrm{sign}(b_{\mathrm{X}^{\mathrm{v}}}) + 1}{2} [F(v,s)]^{q/2} \right)^{1/q}$$

$$F_q^-(s) = \left(\frac{1}{M^-} \sum_{v=1}^{2N_s} \frac{-[\mathrm{sign}(b_{\mathrm{X}^{\mathrm{v}}}) - 1]}{2} [F(v,s)]^{q/2} \right)^{1/q}$$

For $q = 0$, the corresponding expressions become:

$$F_0^+(s) = \exp\left(\frac{1}{2M^+} \sum_{v=1}^{2N_s} \frac{\mathrm{sign}(b_{\mathrm{X}^{\mathrm{v}}}) + 1}{2} \ln[F(v,s)] \right)$$

$$F_0^-(s) = \exp\left(\frac{1}{2M^-} \sum_{v=1}^{2N_s} \frac{-[\mathrm{sign}(b_{\mathrm{X}^{\mathrm{v}}}) - 1]}{2} \ln[F(v,s)] \right)$$

where M^+ refers to subseries with upward trends, while M^-refers to subseries with downward trends. It is assumed that $b_{X_v} \neq 0$ for all v, so that $M^+ + M^- = 2N_s$.

For comparison, the standard MF-DCCA fluctuation function is also computed. For $q \neq 0$, it is given by:

$$F_q(s) = \left(\frac{1}{2N_s} \sum_{v=1}^{2N_s} [F(v,s)]^{q/2} \right)^{1/q}$$

and for $q = 0$:

$$F_q(s) = \exp\left(\frac{1}{4N_s}\sum_{v=1}^{2N_s} \ln\left[F(v,s)\right]\right)$$

Step 5: Scaling behavior and generalized cross-correlation exponents
In order to explore the scaling properties, log-log plots of $F_q(s)$ versus the scale s are constructed for different values of q. If the series are cross-correlated over long ranges, it is expected the fluctuation functions will show a power-law relationship:

$$F_q(s) \sim s^{H_{xy}(q)}$$
$$F_q^+(s) \sim s^{H_{xy}^+(q)}$$
$$F_q^-(s) \sim s^{H_{xy}^-(q)}$$

The exponent $H_{xy}(q)$It is called the generalized cross-correlation exponent and describes the power-law relationship between the two time series. It is estimated by the slope of the log-log plot of $F_q(s)$and s, which is usually obtained by Ordinary Least Squares (OLS).
When $q = 2$, the $H_{xy}(2)$ is analogous to the Hurst exponent in DFA. The value of $H_{xy}(2) > 0.5$, implies cross-persistence (movements in one series tend to be followed by movements in the same direction in the other series). In contrast, $H_{xy}(2) < 0.5$ suggests cross-antipersistence, meaning that movements in one series are more likely to be followed by movements in the opposite direction in the other series. When $H_{xy}(2) = 0.5$, the relationship is short-range dependent or there is no significant cross-correlation.
To quantify the degree of asymmetry in the cross-correlation structure, the following measure is used for each q:

$$\Delta H_{xy}(q) = H_{xy}^+(q) - H_{xy}^-(q)$$

A larger absolute value of $\Delta H_{xy}(q)$indicates a stronger asymmetric effect. If $\Delta H_{xy}(q)$is close to zero, the cross-correlation structure can be considered symmetric across positive and negative trend regimes. A positive value means that the cross-correlation exponent is greater during upward trends in $x_t$than during downward trends, whereas a negative value implies the opposite. The magnitude of $\Delta H_{xy}(q)$reflects the extent of asymmetry in the cross-correlation dynamics. The larger its absolute value, the stronger the asymmetric effect between the two series. If $\Delta H_{xy}(q)$is approximately zero, the cross-correlation pattern can be regarded as symmetric under positive and negative trend conditions of x_t. A positive value indicates that cross-correlations are more pronounced when $x_t$is in an upward trend, while a negative value shows that the cross-correlation exponent is relatively lower during upward trends than during downward trends.

In addition, the dependence of the generalized cross-correlation exponent $H_{xy}(q)$on q provides evidence of multifractality in the relationship between the two series. Similar to the MF-DCCA method, positive values of q emphasize the scaling behavior of large fluctuations, whereas negative values of q highlight the behavior of small fluctuations. Accordingly, this study investigates multifractal dynamics under different trend regimes for both green investments and financial market indices.

3.3.1 Linear Benchmark and Robustness Check

This study employs a linear benchmark analysis, including Pearson and Spearman correlations and an ARDL-type dynamic regression, to assess whether the dependence structure between geopolitical risk and the chosen green and conventional financial assets can be captured by conventional linear methods. While these methods are not alternatives to AMFDCCA, they are used to compare whether the dependence structures detected by the multifractal approach are captured by conventional average-based models. Pearson correlation measures contemporaneous (or linear) dependence, while Spearman correlation measures monotonic dependence. The ARDL-type regression also assesses whether current and lagged values of geopolitical risk capture short-term variations in asset returns.

$$\rho_{i,\mathrm{GPRD}} = \frac{\sum_{t=1}^{T}\left(R_{i,t} - \overline{R}_i\right)\left(\mathrm{GPRD}_t - \overline{\mathrm{GPRD}}\right)}{\sqrt{\sum_{t=1}^{T}\left(R_{i,t} - \overline{R}_i\right)^2}\sqrt{\sum_{t=1}^{T}\left(\mathrm{GPRD}_t - \overline{\mathrm{GPRD}}\right)^2}}$$

Where $R_{i,t}$ is the return of asset i at time t. GPRDt is the geopolitical risk, and T is the sample size. A significant coefficient would suggest that, on average, there is a linear relationship between geopolitical risk and asset returns.

To further examine whether geopolitical risk has short-run dynamic effects on asset returns, an ARDL-type benchmark model is estimated as follows:

$$R_{i,t} = \alpha_i + \sum_{k=1}^{p} \phi_{i,k} R_{i,t-k} + \sum_{j=0}^{q} \beta_{i,j}\,\mathrm{GPRD}_{t-j} + \varepsilon_{i,t}$$

where Ri,t is the return of asset i, Ri,t−k is the lagged return of asset, GPRDt−j is current and lagged geopolitical risk, p and q are the optimal lag lengths, and εi,t is the error term. The lag length is chosen by the Bayesian Information Criterion, and Newey-West robust standard errors are used to correct for heteroskedasticity and autocorrelation. As a linear benchmark and robustness check, this study applies Pearson and Spearman correlations, as well as an ARDL-type dynamic regression. The average linear association of each asset return to GPRD is measured by the Pearson correlation (Pearson, 1895), while the rank-based monotonic dependence of each asset return to GPRD is measured by the Spearman correlation (Spearman, 1904). The benchmark model is an ARDL-type model based on the autoregressive distributed lag model (ARDL) of Pearson and Shin (1999). In this study, however, the model will be applied only to determine short-run linear dynamics and not to measure long-run cointegration. The Bayesian Information Criterion (Schwarz, 1978) is used to determine the optimal lag structure. To account for possible heteroskedasticity and serial correlation in the residuals, Newey–West robust standard errors are applied (Newey & West, 1987).

4. Results and Discussion

4.1 Descriptive Statistics

The descriptive statistics indicate that Bitcoin also has the largest average daily return in the sample (0.0024), with a median of 0.0015. This indicates that Bitcoin outperforms the remaining assets in terms of average daily performance. The ACWI global equity index also shows a positive daily return, but the signs are much smaller (mean = 0.0003, median = 0.0007), reflecting the steady rise in diversified global equities over the period. The mean returns of green investment assets, such as MSCI_SRI, FTSE Good and the Carbon Efficient index, are similarly positive, in the range of about 0.0002 to 0.0003. These values imply slight average gains and are close to the ACWI values. This proximity suggests that the performance of green funds is still highly dependent on the overall equity market. While these funds can include ESG screening or carbon-efficiency tilts, they remain vulnerable to broader market risk sentiment, interest rates, and global economic conditions. It is not possible to consider their average performance without reference to the overall stock market environment. Crude oil and green bonds, on the other hand, exhibit slightly negative mean returns, with values around -0.0001, indicating that negative news events or downward oil prices dominated positive ones in the sample.

The results are applicable to investors seeking steady growth in financial markets across both sustainable and conventional environments. Volatility, the standard deviation, indicates greater variation in returns among assets than the mean returns. Bitcoin has the highest standard deviation, 0.0578, and the largest return range, -0.8488 to 1.4742, making it the most volatile asset. This indicates its volatility and extreme periods of volatility. The standard deviation of crude oil is 0.0293, indicating it is the second most volatile asset, with a minimum of -0.5686 and a maximum of 0.3201. The pattern is expected in commodity markets, which are highly sensitive to macroeconomic developments, supply–demand imbalances, and geopolitical disruptions.

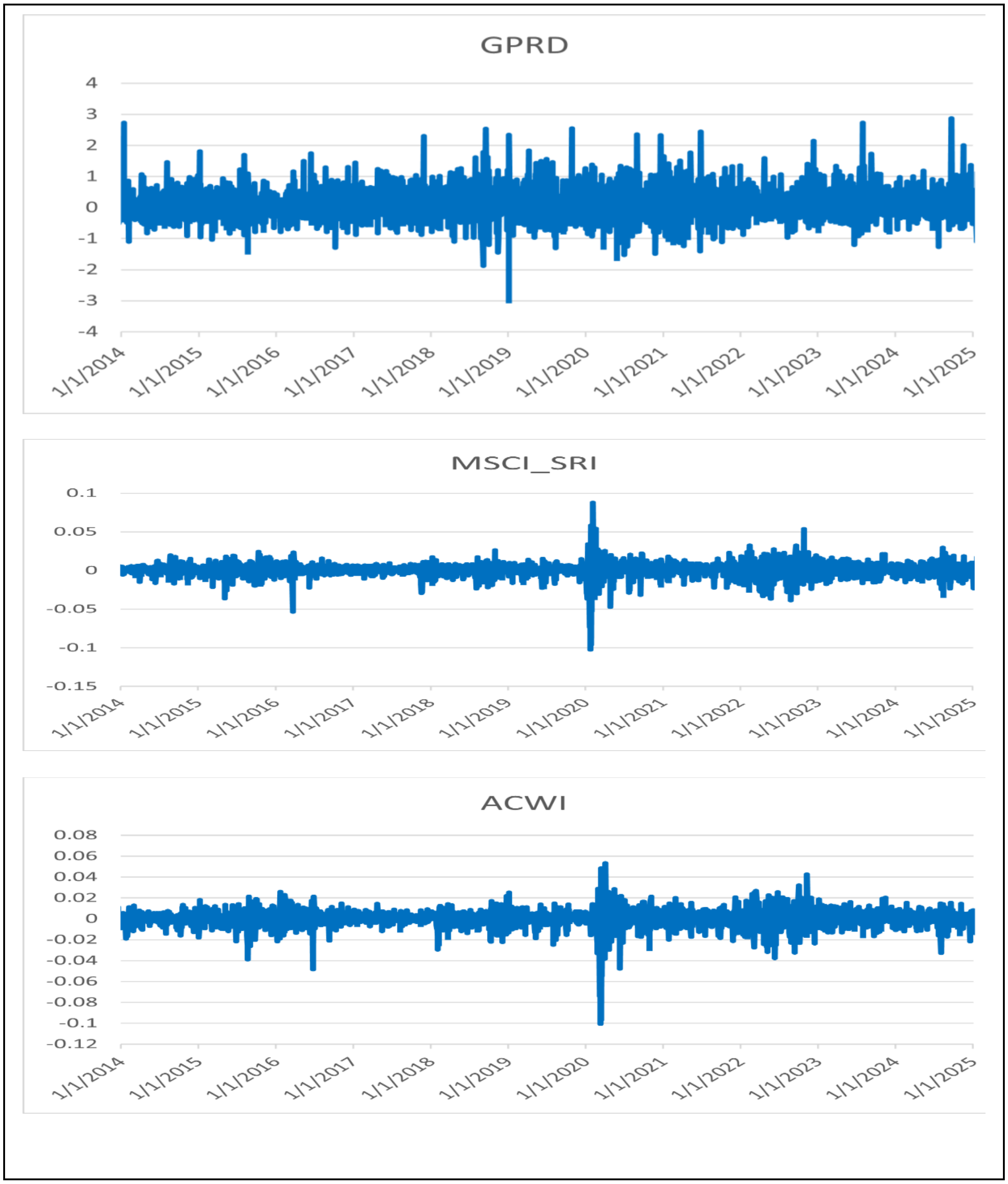
The World Renewable Index and the MSCI Clean Energy Index exhibit relatively high volatility among green investment assets, with standard deviations of 0.0184 and 0.0148, respectively. The volatility of the other broad equity indexes and diversified green funds ranges from 0.009 to 0.010. Gold volatility is in the middle range with a standard deviation of 0.0094, making it less stable than bonds, but much more stable than crude oil and Bitcoin. The most stable return behavior is exhibited by bond indices. The volatility of the SPBOND and green bond series are the lowest with a standard deviation of 0.0033 and 0.0038, respectively, highlighting their stabilizing effect in portfolio construction. This is in line with standard financial theory that bonds are less risky than stocks, commodities or alternative assets. The distributional statistics show that there are large tail risk and asymmetry across the asset classes. All assets, except for bitcoin, have negative skewness, meaning that big negative returns are more common than big positive returns. The downside asymmetry is highest for crude oil, at -2.69, followed by ACWI (-1.4419) and the Carbon Efficient index (-1.3853). From these values, it is clear that the distributions of their returns are strongly left-skewed.

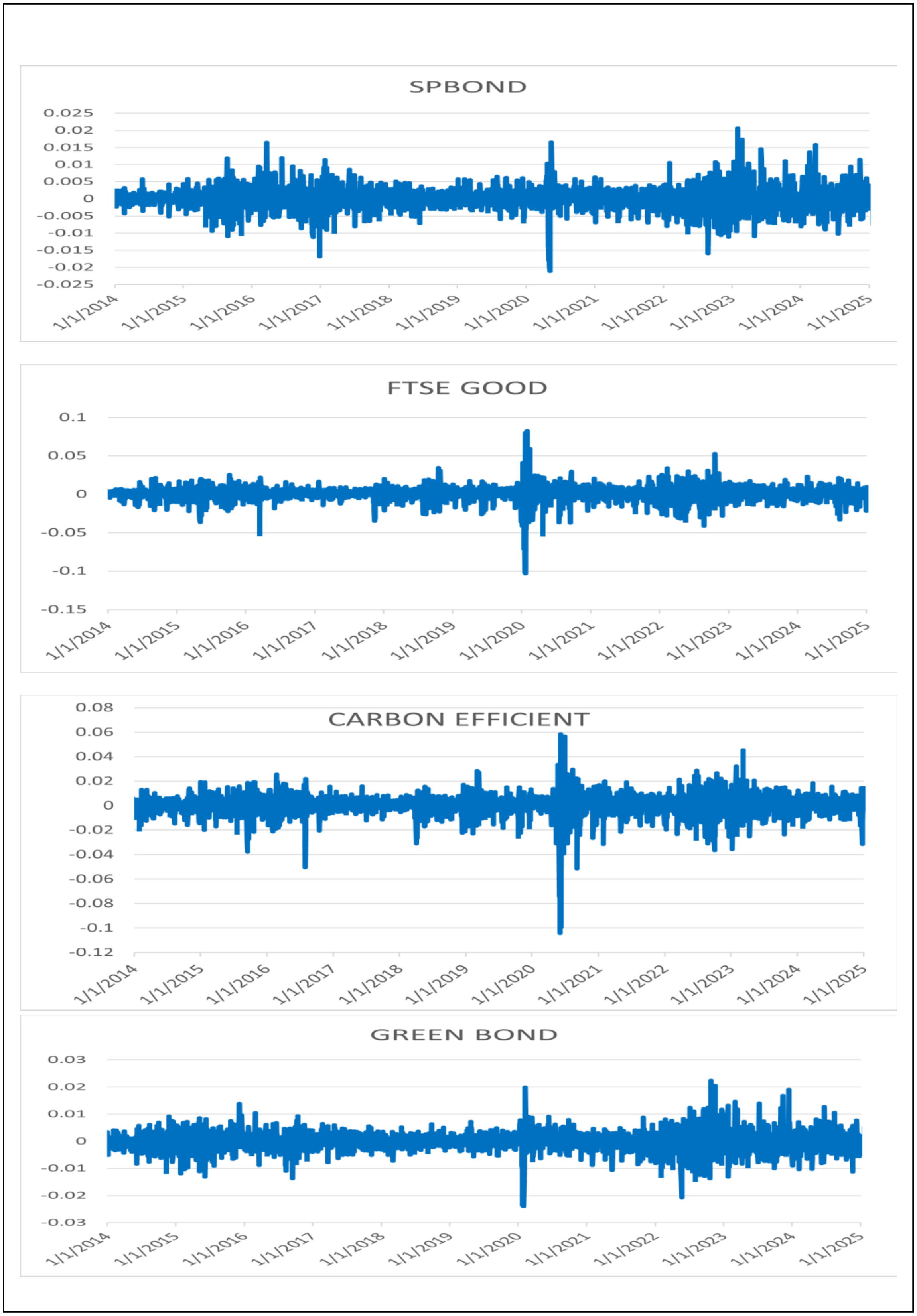
The FTSE Good, MSCI_SRI and Renewable are other green investment assets that display negative skewness (-0.657, -0.7816, -0.7595). The results are significant from a risk-management perspective, as they indicate that negative return shocks are stronger than positive return movements. Bitcoin, however, shows a stronger positive skew of 4.677, which means that there can be some significant positive jumps. But this is not a sign of low risk as Bitcoin has also had significant declines in size. Rather, it reflects the presence of extreme upside outliers alongside high overall uncertainty. The estimates of kurtosis also support the existence of fat-tailed distributions. The kurtosis values for all series are greater than 3, suggesting leptokurtosis and the presence of outliers. Bitcoin and crude oil have high kurtosis values of 175.1163 and 69.6621 respectively, indicating that both assets carry a high degree of tail risk. Kurtosis is also high for equity and green investment assets, especially for the Carbon Efficient index (19.9133) and ACWI (19.3683). This suggests that even broad or sustainability-screened portfolios remain exposed to extreme market movements. In contrast, fixed-income SPBOND (6.9245) and green bonds (6.96) exhibit comparatively lower kurtosis. This relative moderation in tail risk supports using bonds as risk-mitigating instruments in diversified portfolios, particularly when downside tail risk is a primary concern.

Table 2: Descriptive Statistics

	N	Mean	Median	Minimum	Maximum	Standard deviation	Kurtosis	Skewness
BITCOIN	2865	0.0024	0.0015	-0.8488	1.4742	0.0578	175.1163	4.677
GOLD	2865	0.0003	0.0004	-0.0506	0.0553	0.0094	5.8631	-0.2017
SPBOND	2865	0.0001	0.0002	-0.0212	0.0208	0.0033	6.9245	-0.0721
ACWI	2865	0.0003	0.0007	-0.101	0.0538	0.0089	19.3683	-1.4419
CRUDE OIL	2865	-0.0001	0.0011	-0.5686	0.3201	0.0293	69.6621	-2.69
CARBON EFFICIENT	2865	0.0002	0.0006	-0.1045	0.0587	0.0091	19.9133	-1.3853
MSCI_SRI	2865	0.0003	0.0005	-0.1031	0.0882	0.0097	17.9978	-0.7816
RENEWABLE	2865	0.0001	0.0003	-0.224	0.091	0.0184	13.3169	-0.7595
CLEAN ENERGY	2865	0.0001	0.0003	-0.125	0.1103	0.0148	10.6534	-0.3547
FTSE GOOD	2865	0.0003	0.0004	-0.1036	0.0825	0.0101	17.8469	-0.657
GREEN BOND	2865	-0.0001	-0.0001	-0.0242	0.0226	0.0038	6.96	-0.0547
GPRD	2864	0.1015	0.0697	-2.9959	2.8713	0.4935	5.8069	0.5317

The daily returns plots indicate that all green investment funds and financial market indices have a near-zero mean but have significant volatility that concentrates over time. Time-dependent volatility and non-normality of returns suggest time-varying volatility and sharp negative and positive spikes, followed by relatively calm periods. Clean Energy, Renewable, MSCI SRI, FTSE Good, and Carbon Efficient, among the green investment funds, tend to fluctuate during turbulent periods, whereas the Green Bond seems more stable, though it remains exposed to clustered volatility. Bitcoin and crude oil have the most pronounced swings among conventional assets, as they are highly volatile, whereas gold, ACWI, and SPBOND have rather moderate but consistent swings. The geopolitical risk index is volatile, with sudden spikes, indicating the event-driven nature of geopolitical unpredictability. All these trends indicate the presence of heteroskedasticity, asymmetry, and nonlinear dynamics in the return series, hence the use of multifractal techniques to analyze the efficiency and dependence patterns of both green and traditional markets.




SPBOND
0.025
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
1/1/2014 1/1/2015 1/1/2016 1/1/2017 1/1/2018 1/1/2019 1/1/2020 1/1/2021 1/1/2022 1/1/2023 1/1/2024 1/1/2025
FTSE GOOD
0.1
0.05
0
-0.05
-0.1
-0.15
1/1/2014 1/1/2015 1/1/2016 1/1/2017 1/1/2018 1/1/2019 1/1/2020 1/1/2021 1/1/2022 1/1/2023 1/1/2024 1/1/2025
CARBON EFFICIENT
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
1/1/2014 1/1/2015 1/1/2016 1/1/2017 1/1/2018 1/1/2019 1/1/2020 1/1/2021 1/1/2022 1/1/2023 1/1/2024 1/1/2025
GREEN BOND
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1/1/2014 1/1/2015 1/1/2016 1/1/2017 1/1/2018 1/1/2019 1/1/2020 1/1/2021 1/1/2022 1/1/2023 1/1/2024 1/1/2025

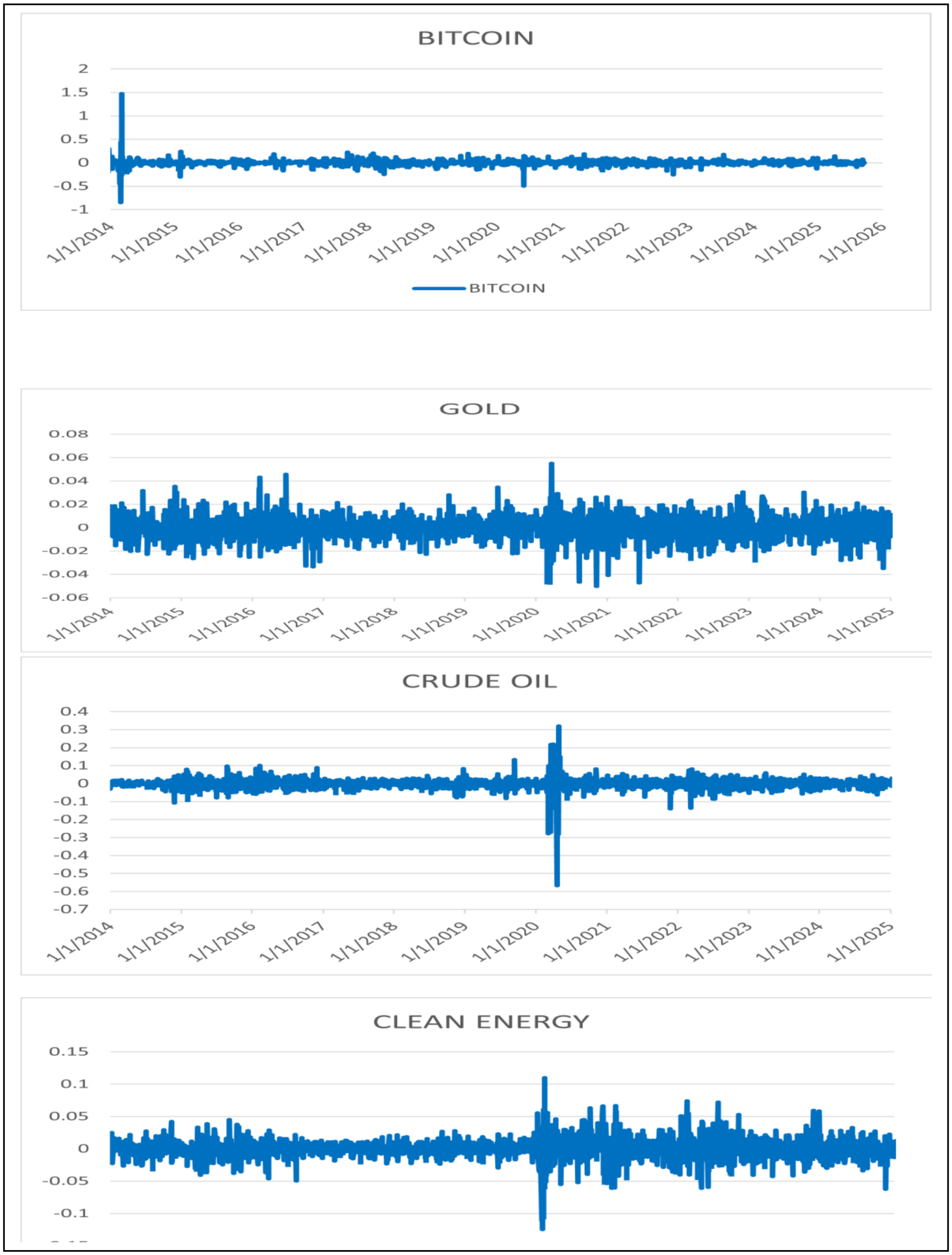
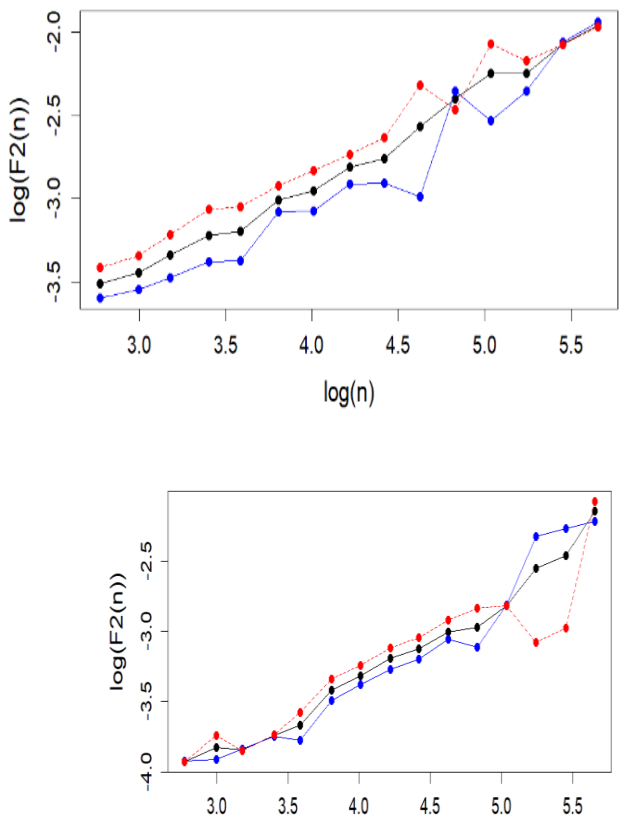


Figure 1: Daily returns of green investment funds, financial markets, and geopolitical risk

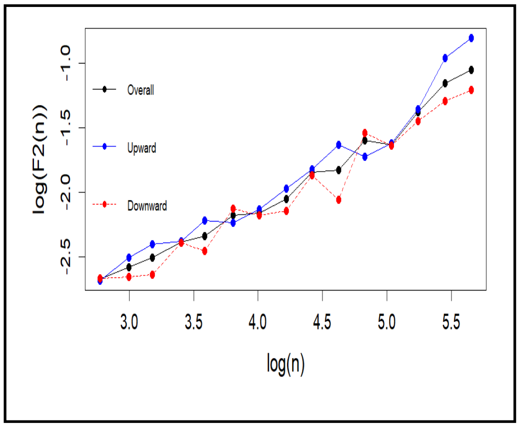
4.2 Multifractal Cross-Correlation Analysis with GPR

4.2.1 Log-log plots of green investment funds, financial markets, and geopolitical risk

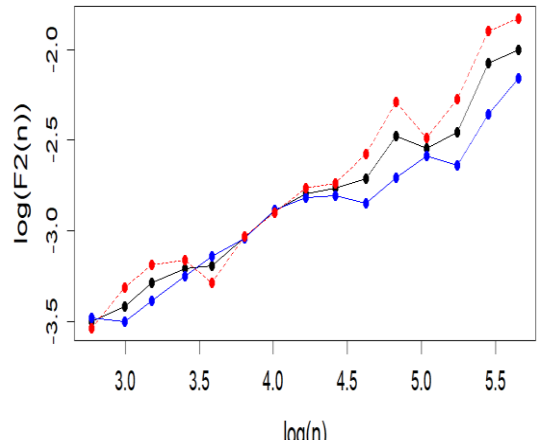
ACWI/GPR



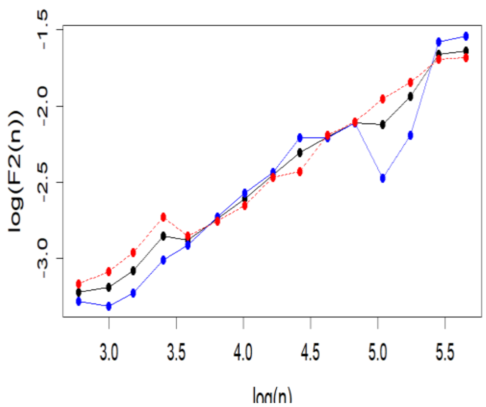
BITCOIN/GPR



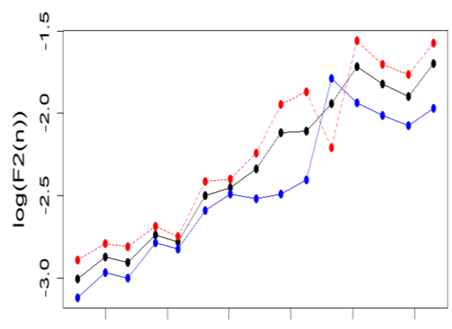
CARBON EFFICIENT/GPR



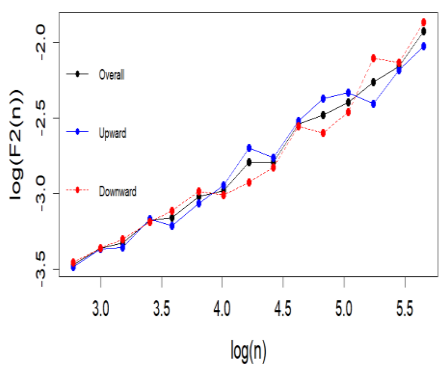
CLEAN ENERGY/GPR



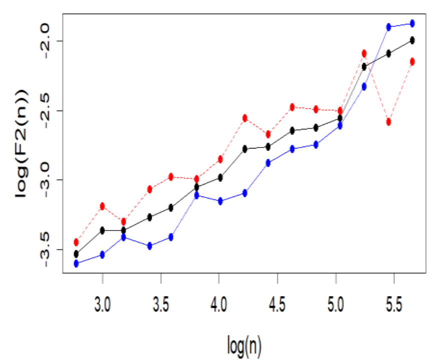
CRUDE OIL/ GPR



GOLD/GPR



FTSE GOOD/GPR



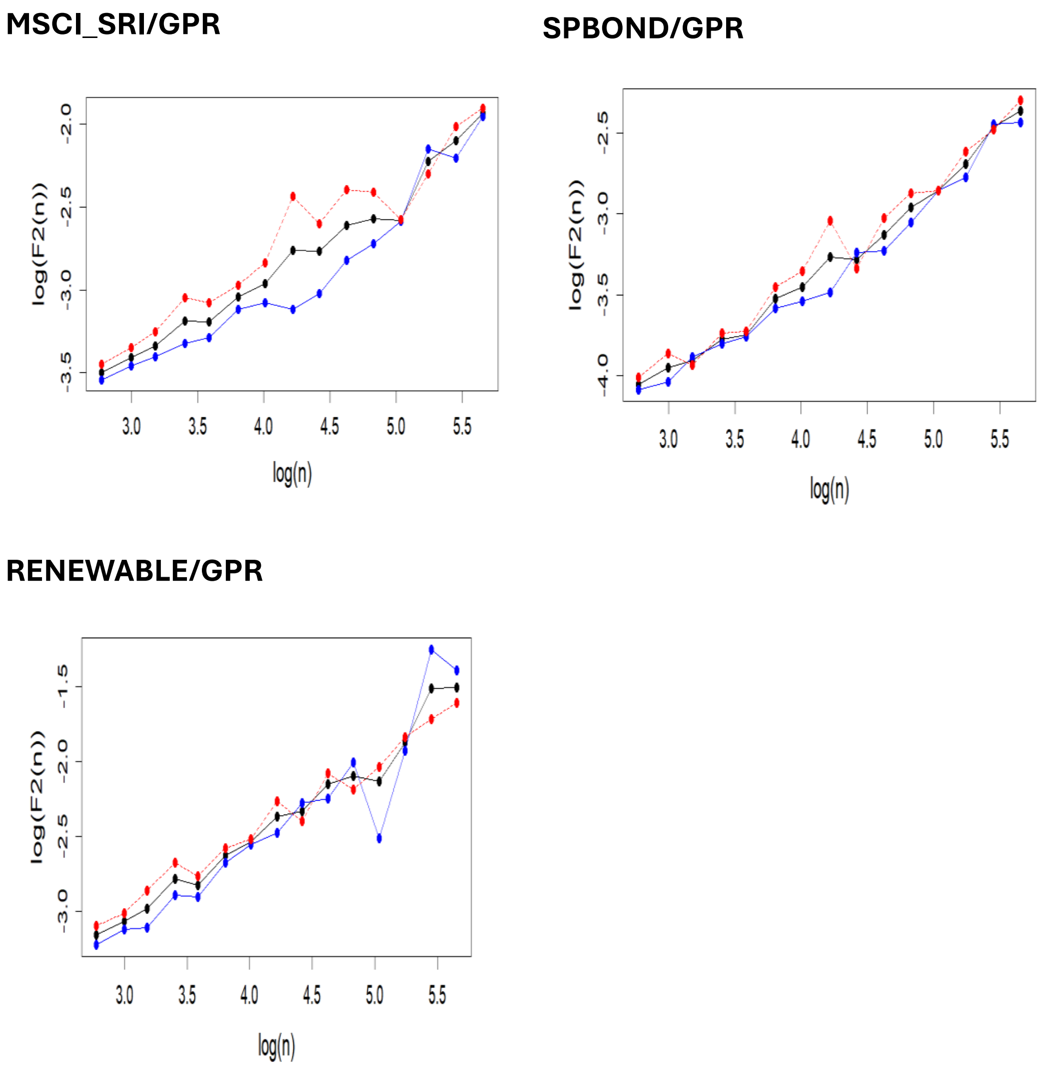


Figure 2: Log-log plots of F2(n), F2+(n), and F2-(n) versus n when the financial markets and green investment funds and geopolitical risk when q ranges from -10 to 10.

The log-log curves of ACWI relative to geopolitical risk are approximately linear across most scales, suggesting stable scaling in the cross-correlations. A noticeable gap appears between the upward- and downward-trend functions, especially at medium and larger scales, indicating asymmetry in the dependence structure. The downward-trend function is generally above the upward one over much of the scale range, suggesting that the cross-correlation exponent tends to be higher when geopolitical risk is declining. This implies that the interaction between ACWI and GPR is relatively stronger under falling GPR conditions than under rising GPR conditions. These findings are consistent with prior studies showing that equity indices exhibit stronger correlations with risky situations during downward trends and asymmetric scaling as suggested by Cao et al., (2014) and Gajardo and Kristjanpoller (2017). Bitcoin exhibits one of the clearest nonlinear, irregular patterns among assets, although the log-log functions still retain an overall scaling tendency. The separation between upward and downward curves is not uniform across all scales, which suggests a more heterogeneous asymmetric structure. At larger scales, the upward-trend function tends to dominate, indicating that Bitcoin becomes more strongly cross-correlated with GPR as geopolitical risk increases. This is consistent with Bitcoin's more unstable, fluctuation-sensitive nature and suggests that the asymmetric response is more pronounced during stronger GPR episodes. The heightened association between Bitcoin and rising GPR is consistent with findings that Bitcoin responds strongly to uncertainty events, while its hedging ability is unstable and state-dependent (Bouri et al., 2017; Aysan et al., 2019).

The Carbon Efficient index shows relatively smooth log-log scaling, with the three curves moving closely together at small scales and separating more clearly as the scale increases. The downward-trend function lies above the upward one over most of the range, indicating stronger cross-correlations when GPR is declining. However, compared with assets such as Gold, Bitcoin, Renewable energy, clean energy, and green bonds, the asymmetry appears moderate rather than extreme. This indicates that the Carbon Efficient Index has a relatively more stable dependence structure with geopolitical risk. However, the clean energy plot shows a clear log-log relationship, thereby supporting the presence of long-range cross-correlation with geopolitical risk. However, the asymmetry becomes more visible at medium and large scales, where the upward and downward fluctuation functions diverge. The upward curve overtakes the downward one toward the higher scales, implying that the generalized cross-correlation exponent is larger when GPR is upward.

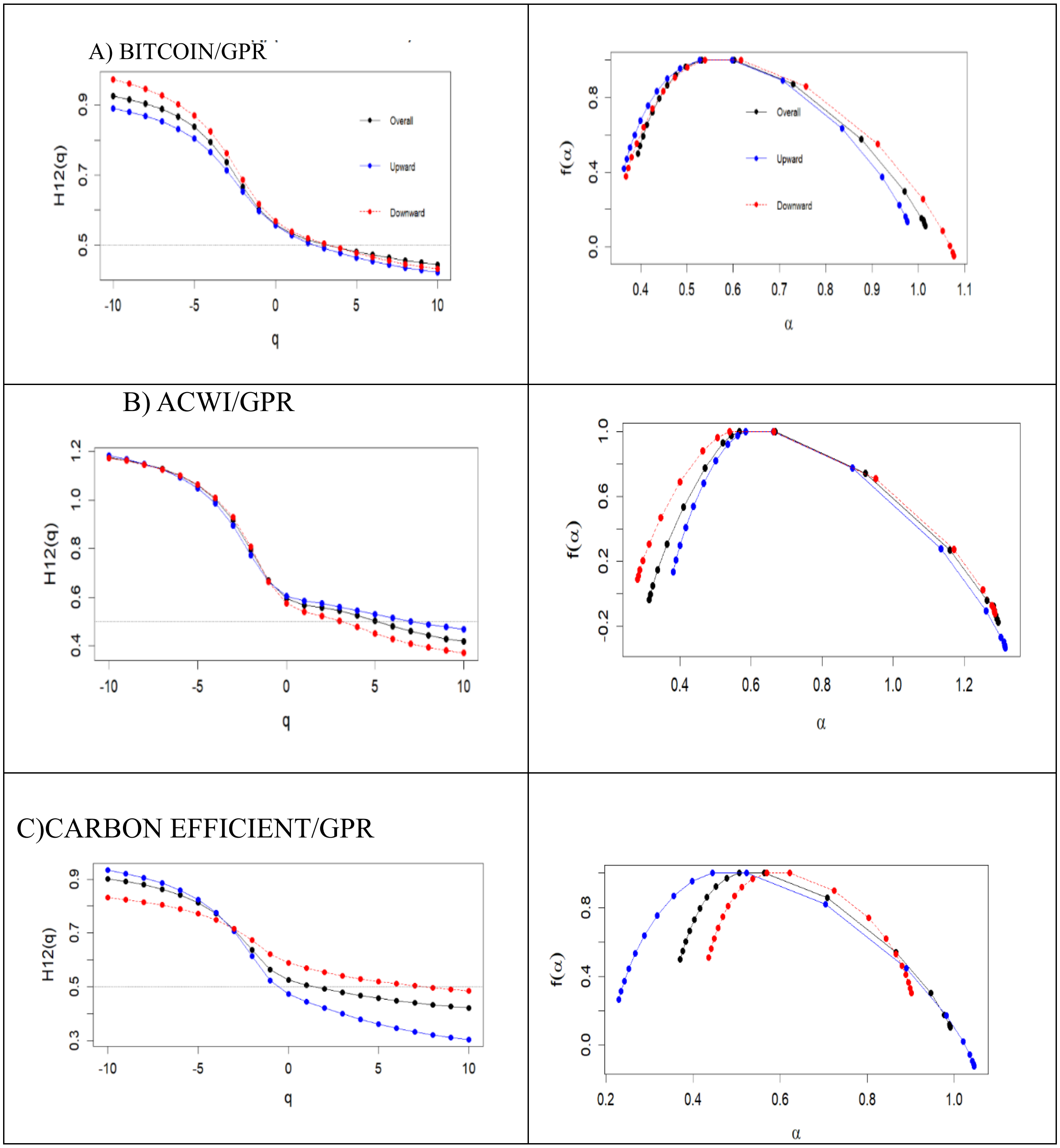
This indicates that clean energy assets become more tightly linked to geopolitical risk due to their exposure to energy transition policies, investor sentiments, and global risk conditions. Similar results were observed by Ghaemi Asl et al., (2025), who report that crises intensify multifractal dependencies between green assets and energy markets, especially during periods of heightened uncertainty. Crude oil exhibits a strong and visibly asymmetric pattern. The downward-trend function lies above the upward one for most scales, and the separation is relatively large compared with several green assets. This indicates that the crude oil–GPR cross-correlation exponent is higher when geopolitical risk declines than when it increases. In other words, the oil market's linkage with GPR appears stronger under declining GPR trends, although the overall relationship remains clearly persistent and scale dependent. The highly asymmetric response of oil is consistent with previous studies demonstrating that energy markets are sensitive to geopolitical risk due to supply-side issues and uncertainty expectations (Antonakakis et al., 2017; Sarker et al., 2023; Zhang et al., 2023; Ren et al., 2023). The FTSE Good index exhibits stable scaling and a clear separation between the upward and downward trend functions. The downward-trend fluctuation function remains above the upward-trend fluctuation function across most scales, suggesting stronger cross-correlation when GPR is falling. The difference is especially noticeable at medium scales, which indicates moderate but consistent asymmetry. This indicates that ESG-oriented broad equity performance is more strongly connected to GPR under downward geopolitical risk conditions. Gold exhibits one of the most stable patterns, as the upward and downward movements remain closer together than for several other assets. A slight asymmetry remains evident, especially at larger scales, where the downward trends are marginally above the upward trend toward the end of the range. This suggests that gold's cross-correlation with GPR is asymmetric, although less pronounced than for Bitcoin or crude oil. Gold shows a long-range cross-correlation with only limited asymmetry across different GPR trends. Gold has shown persistent but relatively weak asymmetry, consistent with previous evidence that gold is often a hedge asset, though hedge performance varies across market regimes (Baur and Lucey, 2010; Reboredo, 2013).

The green bond index shows approximately linear log-log scaling, which is consistent with long-range correlation with geopolitical risk. The separation between upward and downward trends widens at larger scales, where the upward and overall curves rise above the downward one. This indicates that the cross-correlation exponent becomes larger when GPR is upward, especially in the high-scale region. Thus, green bonds appear to react more strongly to rising geopolitical uncertainty than to falling geopolitical risk, which is a clear asymmetric pattern. The MSCI SRI index shows persistent scaling and a noticeable asymmetry between the two conditional fluctuation functions. Across the scale range, the downward trend remains above the upward trend function, indicating a stronger cross-correlation structure under downward GPR trends. However, the difference between the upward and downward curves is more pronounced at the largest scales, while the overall trend remains the same, with greater dependence during periods of falling geopolitical uncertainty.

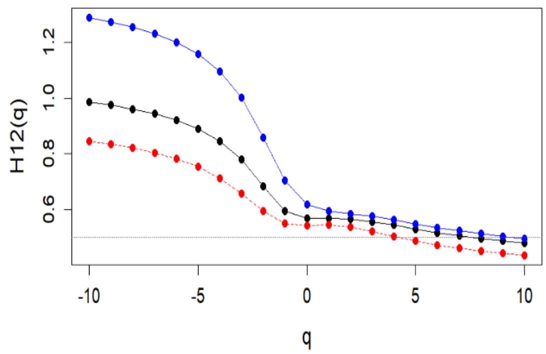
The renewable-energy index exhibits unstable scaling behavior at medium-to-large scales, reflecting a more heterogeneous multifractal structure. However, the upward-trend function grows significantly at the largest scales and eventually surpasses the downward function, indicating that renewable assets have a stronger association with GPR at longer time horizons. Moreover, green bonds, clean energy, and renewable assets were more strongly linked as GPR increased, supporting recent findings that sustainable financial markets are more connected during crises (Naeem et al., 2024). The SPBOND index shows one of the clearest linear log-log patterns, indicating a stable long-range cross-correlation structure. The downward-trend function is slightly above the upward one over much of the range, though the gap is not extremely large. This suggests mild asymmetry, with stronger bond-market cross-correlation under downward GPR trends. Compared with more volatile assets, the asymmetry in SPBOND-GPR appears to be more moderate and uniform across scales.

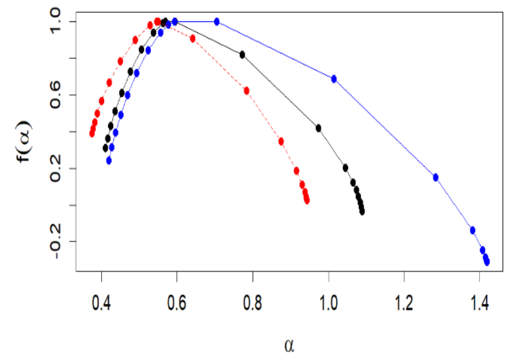
Overall, these results indicate that geopolitical risk generates asymmetric, scale-dependent cross-correlations across all selected assets, but the direction and strength of the asymmetry differ across assets. Clean energy, Renewable energy, Green bonds, and Bitcoin show stronger linkages amid rising geopolitical risk, especially on larger scales. By contrast, ACWI, Carbon Efficient, Crude oil, FTSE Good, MSCI SRI, and SPBOND show greater dependence during periods of declining geopolitical risk. Gold appears relatively stable in the log-log plot, but this visual pattern should be interpreted alongside the Hurst exponent and multifractal spectrum results, which reveal stronger scale-dependent dynamics. These findings suggest that green investment funds are heterogeneous and should not be analyzed as a single uniform asset class in response to geopolitical risk.

4.2.2 Hurst Exponent and Multifractal Spectrum

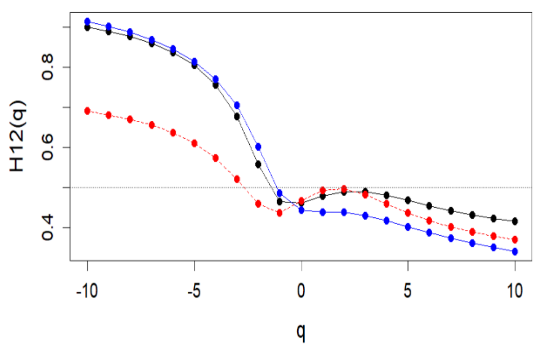


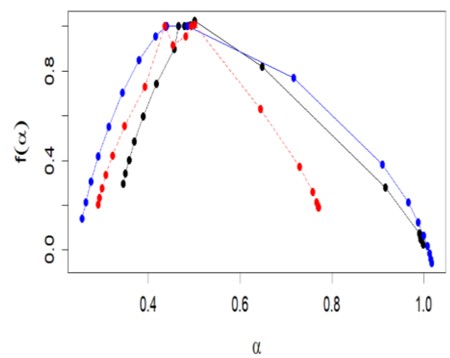
D) CLEAN ENERGY/GPR



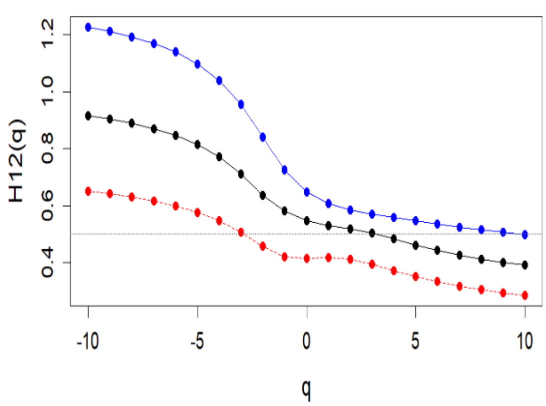


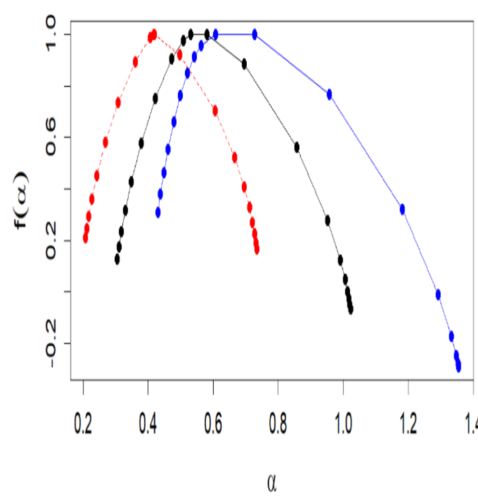
E) CRUDE OIL/GPR



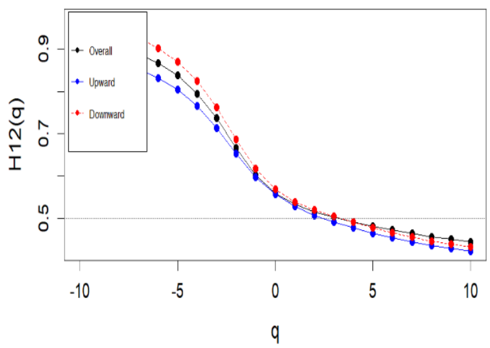


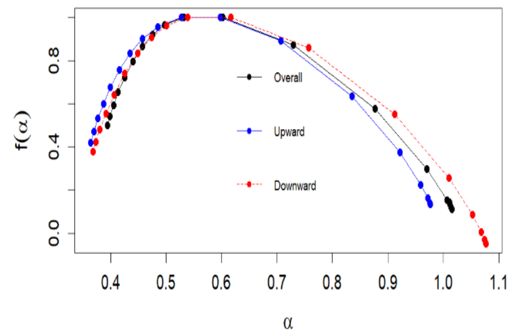
F) FTSE GOOD/GPR



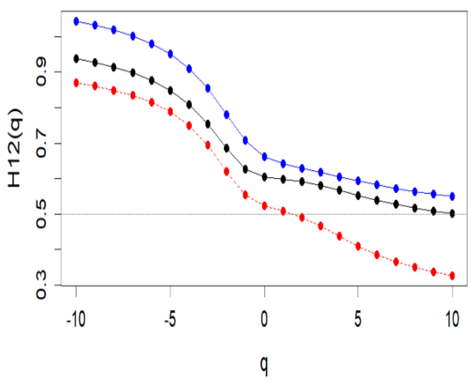


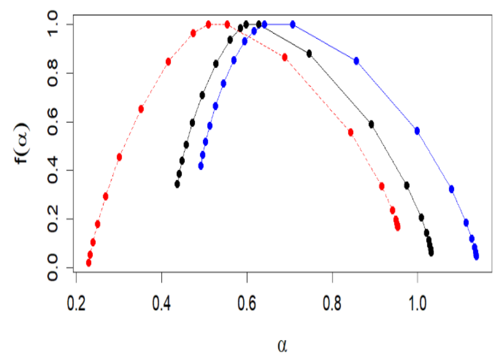
G) GOLD/GPR



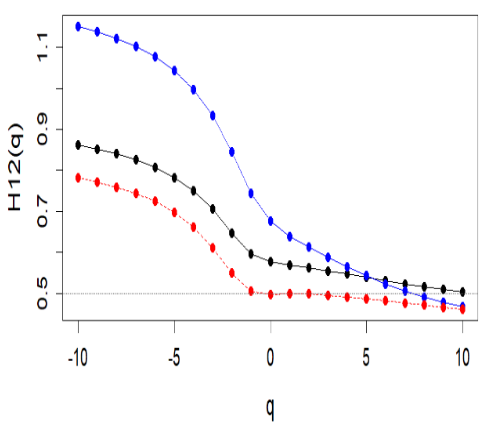


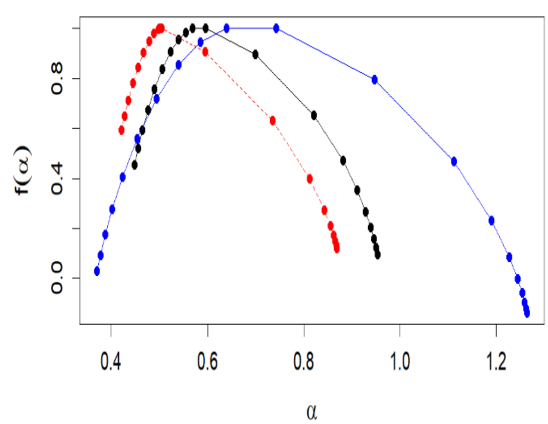
H) GREEN BONDS/GPR





I) RENEWABLE/GPR





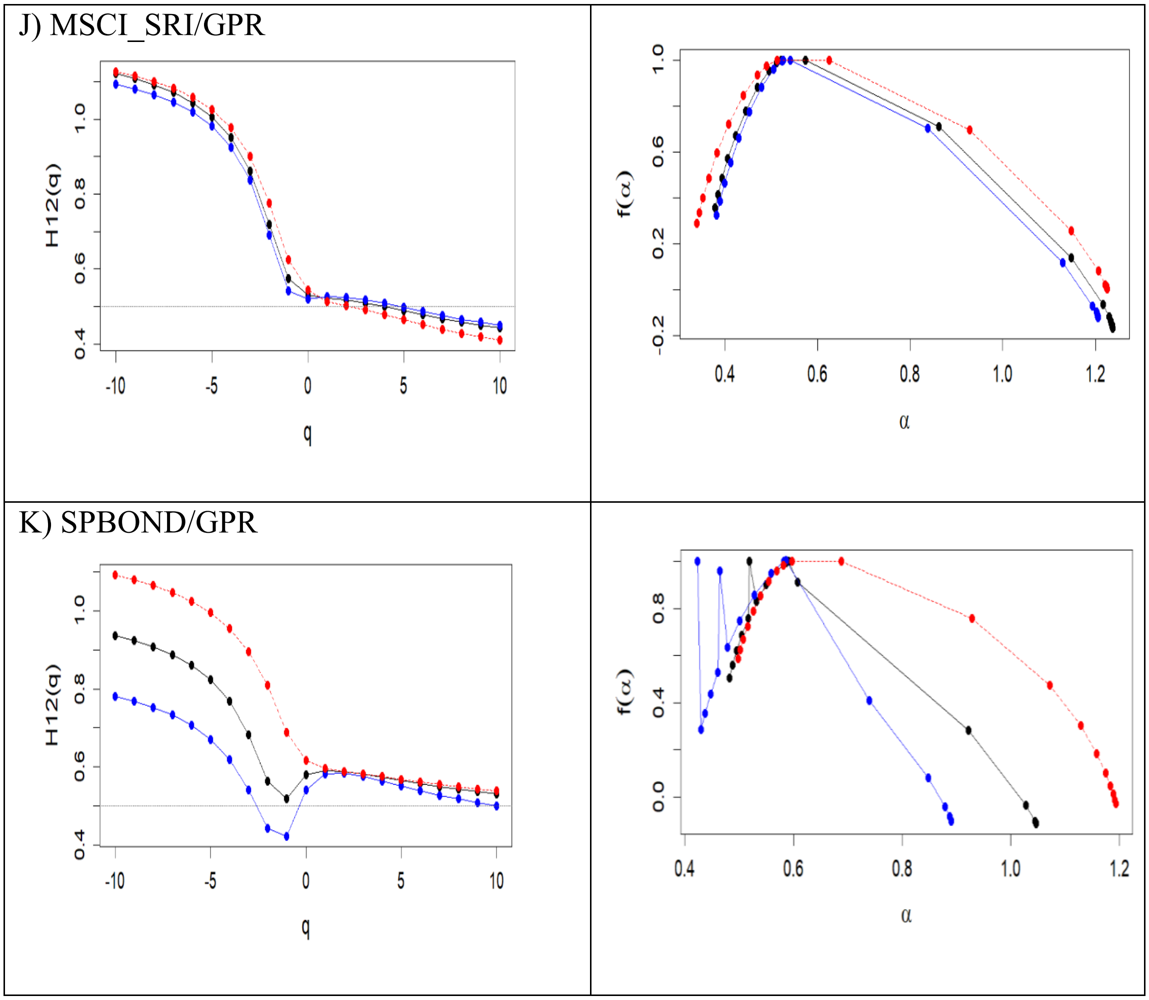


Figure 3: Hurst exponent and Multifractal spectrum plots for green investment funds, financial markets, and geopolitical risk

The ACWI/GPR pair exhibits evidence of asymmetric multifractal correlation. The generalized Hurst exponents vary significantly with the value of q, suggesting non-monofractal dependence structure. For negative values of q the exponents are still clearly above 0.5, indicating that there is still a significant cross-correlation during small fluctuations. The exponents gradually decrease towards 0.5 and at higher orders they tend to approach or get just below 0.5 as q increases into the positive range. This implies that persistence weakens as the size of fluctuations increases. Moreover, the upward and downward exponents are unequal, confirming asymmetry, as suggested by Cao et al. (2014). Since the upward curve becomes more dominant at higher positive q, the ACWI–GPRD linkage seems more persistent under rising GPRD conditions for large fluctuations. The broad and slightly right-skewed spectrum further suggests a pronounced multifractal structure with stronger influence from smaller-amplitude fluctuations (Kantelhardt et al., 2002). Moreover, the Bitcoin–GPRD pair also shows multifractal asymmetric cross-correlation, because the generalized Hurst exponents are clearly q-dependent across the fluctuation spectrum.

The exponents are above 0.5 for negative and intermediate q, indicating persistent cross-correlation for small and moderate fluctuations, but they approach 0.5 at large positive start equation q, indicating weaker persistence of large fluctuations. A notable feature is that the downward curve remains above the upward curve across most of the q-range, which, under the AMFDCCA interpretation of $\Delta H_{12}(q)$, indicates that the Bitcoin–GPRD relationship is more persistent when GPRD is falling than when it is rising. Such findings are in line with Gaies et al., (2025), which examined the relationship between mining activity and climate policy uncertainty and report time-varying and asymmetric interrelations, indicating that cryptocurrencies are highly sensitive to policy and environmental uncertainty. The fairly broad but comparatively compact spectrum implies moderate-to-strong multifractality, while the slight right-skewness suggests a stronger contribution from smaller fluctuations (Zhang et al., 2019).

However, the Carbon Efficient–GPRD pair exhibits scale-dependent asymmetry. The strong dependence of $H_{12}^{+}(q)$and $H_{12}^{-}(q)$ on q confirms multifractality, while the sign change in the relative position of the two curves indicates that the asymmetry itself is scale contingent. For negative q, the upward exponent exceeds the downward exponent, implying stronger persistence under rising GPRD for small fluctuations. For positive qHowever, the order reverses, so that the downward regime becomes more persistent for large fluctuations. This pattern is highly consistent with the (Fan et al., 2019; Lee et al., 2020), where the evolution of $\Delta H_{12}(q)$across q is used to distinguish whether asymmetry is stronger for small or large fluctuations. The broad spectrum and visible trend separation reinforce the conclusion that the Carbon Efficient–GPRD nexus is characterized by pronounced asymmetric multifractality. The Clean Energy/GPRD relationship exhibits one of the strongest multifractal patterns among the green investments.

The sharp decline of the generalized Hurst exponents across q indicates substantial heterogeneity in scaling behavior, which is a hallmark of multifractality. For negative q, the upward exponent is far above the downward one, implying very strong persistence for small fluctuations when GPRD is rising. Although the exponents fall as q increases and approach 0.5 for large fluctuations, the upward trend remains dominant across most of the range, indicating that the cross-correlation is more persistent under rising geopolitical risk. The wide spectrum reflects strong multifractality, and the right skewness suggests that the small and medium fluctuations contribute more strongly to the multifractality than large shocks (Memon et al., 2023. Naeem et al., 2022).

The Crude Oil/GPRD pair displays significant asymmetric multifractal cross-correlation. The variation in generalized Hurst exponents across $q$confirms heterogeneous scaling behavior, implying that the oil–GPRD nexus varies with fluctuation size. For negative q, the upward regime is more persistent, indicating stronger memory for small fluctuations during rising GPRD. For positive q, this dominance weakens and partly reverses, suggesting that the downward regime becomes comparable to or slightly more persistent for large fluctuations. This scale-dependent reversal is consistent with the AMFDCCA framework and parallels the crude oil multifractality findings by (Gu et al., 2010; Gaio & Capitani, 2023; Ramirez et al., 2002), which show that oil markets exhibit distinct persistence properties for small-versus large-scale fluctuations. The broad singularity spectrum confirms strong multifractality, and the broader upward spectrum suggests greater structural complexity as GPRD rises.

Other indices, including FTSE GOOD, Gold, Green bonds, MSCI_SRI, Renewable, and SPBOND exhibit multifractal asymmetric cross-correlation. The geopolitical risk does not transmit uniformly across financial and green investment funds. The pronounced variation of the generalized Hurst exponents across q confirms that the dependence structure is scale-dependent rather than monofractal, while the separation between the upward and downward conditional exponents indicates that the relationship changes depending on whether geopolitical risk is rising or falling. Across the assets, persistence is generally stronger for small fluctuations and becomes weaker for large fluctuations as q shifts from negative to positive values.

Nevertheless, the nature of this asymmetry is not uniform across markets. FTSE Good, Green Bond, and Renewable Energy show stronger persistence when GPRD is rising, suggesting that these assets are more sensitive to intensifying geopolitical uncertainty. The visual patterns in the Hurst exponent plots for Gold are relatively stable, and its ΔH12 value suggests strong scale-dependent multifractality. This suggests that Gold's dependence on geopolitical risk changes substantially between small and large fluctuations, even the visual separation between upward and downward trends is less irregular than Bitcoin or Renewable Energy. The persistence pattern of MSCI_SRI is relatively well-balanced and scale-dependent, with more persistence on small up and down movements. On the other hand, the dominance of the downward trend is clearly the strongest in SPBOND. The overall findings suggest that geopolitical risk has a nonlinear and asymmetric relationship with these indices. Cross correlations intensify and deepen as geopolitical risk rises, for most sustainability-focused assets. This implies that geopolitical uncertainty is not transmitted evenly across financial markets. Some indices are more responsive to increases in GPR, whereas others, such as gold and bonds, show more stable or even counterintuitive patterns (Aslam et al., 2022).

The empirical results also indicate that the impact of geopolitical risk varies by asset class and is not equivalent to green and conventional markets. In terms of absolute values of ΔH_{12}, the most scale-dependent asymmetric multifractal character is observed for Gold, followed by Renewable Energy, Bitcoin, Clean Energy, and Green Bonds. This ranking illustrates that these assets have the widest range of persistence from small to large fluctuations. In comparison, MSCI_SRI, FTSE Good, and Carbon Efficient have lower ΔH_{12} values, indicating relatively less volatility in the dependence structure with respect to geopolitical risk across various scales. Between these two groups is another group consisting of ACWI, SPBOND and Crude Oil, with clear but less pronounced asymmetric multifractality. The directional asymmetry also differs across markets. Clean energy, Renewable energy, Green bonds, and Bitcoin show stronger linkages to geopolitical risk under rising GPR conditions, particularly at medium and large scales. This suggests that these assets become more sensitive when geopolitical risk intensifies. By contrast, ACWI, Carbon Efficient, Crude oil, FTSE Good, MSCI_SRI, and SPBOND generally show stronger cross-correlation under declining GPR conditions. Gold displays strong scale-dependent multifractality, but its upward and downward conditional patterns are comparatively more stable than highly fluctuation-sensitive assets such as Bitcoin and Renewable energy. These results confirm that green financial assets should not be treated as a homogeneous asset class because their sensitivity to geopolitical risk varies by asset type, market function, and the scale of fluctuations.

The AMFDCCA findings provide partial acceptance for H1, as the results confirm that dependence with geopolitical risk is asymmetric and scale-dependent for green investment assets. But sensitivity varies across green assets, and not all indices exhibit the same sensitivity. H_2 is also supported, as the findings show that the dependence is not symmetric between rising and falling GPR, and that the direction of stronger dependence varies across assets. While some assets are more strongly linked to increasing GPR, others are more strongly or more persistently linked to decreasing GPR. H3 is supported because the results demonstrate that geopolitical risk is linked with nonlinear, asymmetric, and multifractal cross-correlations between green and conventional financial markets.

Table. 3 presents the generalized Hurst exponent values $H_{xy}(q)$for different asset pairs with respect to GPR across a range of q values from -10 to 10. These values reflect how the cross-correlation between each asset and GPRD varies across different fluctuation scales, with negative q values representing smaller fluctuations and positive q values representing larger fluctuations. In the AMFDCCA analysis, we used a range of q-orders from −10 to +10, following Cao et al. (2014) and it is methodologically important, as negative values of q emphasize the scaling of small fluctuations, whereas positive values of q emphasize large fluctuations. Therefore, this range provides a comprehensive overview of the asymmetric multifractal cross-correlations between upward and downward market movements. A narrower range of q-values may not capture the full fluctuation structure, especially for financial series where both large and small fluctuations contain valuable information. This range is also in line with previous multifractal analysis of financial markets as studied by Aslam et al., (2022), which consider nonlinear dependence, persistence, and asymmetry of fluctuations of various sizes.

A key finding from the table is the general asymmetry in the relationship between these asset pairs and GPRD: for most asset pairs, $H_{xy}(q)$is higher for negative q, indicating stronger persistence during small fluctuations, and decreases as $q$increases, suggesting that the relationship weakens for large fluctuations. In other words, geopolitical risk appears to have stronger long-range dependence with most assets during relatively smaller or normal market movements, whereas this dependence becomes weaker or less stable during larger fluctuations.

The ΔH_{12} values provide a more accurate comparison of scale-dependent multifractality between the assets considered. When considering the scale-dependent multifractality in absolute terms, the highest value is for Gold with -0.74, followed by Renewable Energy (-0.69), Bitcoin (-0.58), Clean Energy (-0.55), and Green Bonds (-0.49). These findings suggest that the largest changes occur between small and large fluctuation dynamics in these assets, meaning their dependence on GPRD can change significantly with market conditions. The Gold asset pair exhibits the most pronounced scale-dependent multifractal behavior, as evidenced by the steep decrease in Hxy(q) with increasing q. This implies that the cross-correlation between Gold and GPRD is also asymmetric, with a higher degree of correlation in smaller fluctuations than in larger ones. The finding aligns with the views of Moreira et al. (2025), who suggest that gold exhibits anti-persistence and multifractality at higher levels of geopolitical risk. Their evidence also suggests that the safe-haven role of gold is not static but rather varies with market conditions and the magnitude of volatility.

Bitcoin and Renewable energy also show strong scale-dependent multifractality. Their $H_{xy}(q)$ values are relatively high for negative q but drop as q approaches positive values. This pattern suggests that they are more strongly linked to GPRD in smaller fluctuations and less linked in larger fluctuations. In the case of Bitcoin, this behavior is particularly important because it differs from that of traditional financial indices. It has a relatively high ΔH_{12}, indicating that the relationship is unstable and sensitive to fluctuations in geopolitical risk. The multifractal jump dynamics and greater sensitivity to negative geopolitical news reported by Ali et al. (2024) for Bitcoin further substantiate this interpretation, as the asymmetric and anti-persistent behavior observed in the examples with larger fluctuations is also evident during smaller fluctuations.

Renewable Energy, Clean Energy, and Green Bonds, among green investment assets, exhibit the most pronounced scale-dependent behavior. This indicates that these assets are more susceptible to geopolitical risk than other ESG-related indexes. Renewable Energy and Clean Energy might be particularly vulnerable, as they are closely linked to energy-transition expectations, policy uncertainty and investor confidence in low-carbon technologies. Green Bonds also exhibit distinct multifractal characteristics: high cross-correlations in smaller fluctuations, and lower cross-correlations in larger fluctuations. This finding is consistent with the research by Naeem et al. (2021) which revealed that Green Bonds are multifractal in nature and are influenced by external risk factors. In contrast, Carbon Efficient, FTSE Good, and MSCI_SRI exhibit weaker scale-dependent multifractality, with smaller variations in $H_{xy}(q)$ over the q range and $H_{xy}(q)$ values close to 0.5. This indicates that their relationship with GPRD is more stable across different fluctuation sizes and less dependent on the scale of market movements.

Crude Oil, ACWI, and SPBOND show moderate multifractality. While the relationship with GPRD is still persistent for small fluctuations, the persistence drops as $q$increases. This suggests that these assets, although still sensitive to geopolitical risk, exhibit a more stable and less asymmetric response compared to assets like Gold and Bitcoin. In the case of Crude oil, the moderate multifractal behavior remains economically meaningful because oil markets are directly linked to geopolitical tensions, supply-side uncertainty, and global energy security. Aslam et al. (2022) also confirm that multifractal cross-correlation between geopolitical risk and commodities such as WTI is stronger during smaller fluctuations.

Overall, Table. 3 confirms that geopolitical risk has a heterogeneous and scale-dependent impact on financial and green investment assets. Asset pairs like Gold, Bitcoin, and Renewable energy, Clean energy and Green bonds show strong, asymmetric multifractal cross-correlation with GPR. On the other hand, FTSE Good, MSCI_SRI, and Carbon Efficient display weaker multifractal responses, indicating that their relationship with GPR is less sensitive to changes in fluctuation scale. The varying degrees of asymmetry and persistence observed across asset pairs underscore the heterogeneous response of markets to geopolitical risk. These findings show that green assets should not be treated as a single uniform category. Renewable energy, Clean energy, and Green bonds are more fluctuation sensitive, whereas broader ESG-oriented indices such as MSCI_SRI, FTSE Good, and Carbon Efficient behave more like diversified equity indices. The varying degrees of persistence and multifractality across asset pairs support the argument that geopolitical risk transmission is nonlinear, scale-dependent, and asset-specific.

Table.3 Hurst exponent for green investment funds, financial markets and geopolitical risk ranging from q = -10 to +10

q	ACWI/ GPRD	BITCOIN /GPRD	CARBON EFFICIENT /GPRD	CLEAN ENERGY /GPRD	CRUDE OIL /GPRD	FTSE GOOD	GOLD /GPRD	GREEN BOND/ GPRD	MSCI _SRI/ GPRD	RENEWABLE /GPRD	SPBOND /GPRD
-10	0.99	1.04	0.59	1.08	0.9	0.5	1.19	1.01	0.67	1.21	0.86
-9	0.97	1.03	0.58	1.07	0.89	0.49	1.18	1	0.66	1.19	0.85
-8	0.96	1.02	0.57	1.05	0.88	0.48	1.16	0.98	0.65	1.18	0.84
-7	0.94	1	0.56	1.03	0.86	0.47	1.13	0.96	0.63	1.15	0.82
-6	0.91	0.98	0.54	1.01	0.83	0.46	1.1	0.94	0.61	1.12	0.8
-5	0.87	0.96	0.52	0.97	0.8	0.44	1.05	0.9	0.57	1.08	0.78
-4	0.81	0.92	0.49	0.91	0.75	0.42	0.98	0.85	0.52	1.02	0.74
-3	0.72	0.86	0.45	0.81	0.67	0.39	0.87	0.77	0.44	0.91	0.68
-2	0.59	0.74	0.42	0.68	0.57	0.37	0.71	0.66	0.35	0.74	0.61
-1	0.5	0.56	0.44	0.57	0.5	0.42	0.58	0.56	0.38	0.57	0.55
0	0.5	0.52	0.47	0.55	0.5	0.48	0.53	0.53	0.46	0.53	0.53
1	0.52	0.5	0.47	0.55	0.52	0.5	0.51	0.53	0.48	0.52	0.52
2	0.54	0.5	0.46	0.55	0.55	0.5	0.5	0.55	0.5	0.53	0.51
3	0.57	0.49	0.45	0.55	0.58	0.49	0.5	0.56	0.52	0.53	0.5
4	0.59	0.49	0.44	0.56	0.6	0.47	0.49	0.57	0.53	0.53	0.5
5	0.61	0.49	0.44	0.55	0.6	0.45	0.49	0.56	0.54	0.53	0.5
6	0.61	0.48	0.43	0.55	0.59	0.43	0.48	0.55	0.54	0.53	0.49
7	0.61	0.48	0.42	0.54	0.58	0.41	0.47	0.54	0.54	0.53	0.49
8	0.6	0.47	0.41	0.54	0.57	0.39	0.47	0.53	0.54	0.52	0.49
9	0.6	0.47	0.41	0.53	0.57	0.37	0.46	0.52	0.54	0.52	0.48
10	0.59	0.46	0.4	0.53	0.56	0.36	0.45	0.51	0.54	0.51	0.48
ΔH12	-0.39	-0.58	-0.19	-0.55	-0.34	-0.14	-0.74	-0.49	-0.13	-0.69	-0.38

4.2.3 Robustness Check

Table 4 indicates that the Pearson and Spearman correlation coefficients show limited contemporaneous association between GPRD and the green and conventional financial assets. The coefficients are small and close to zero for all assets, indicating that GPRD does not exhibit a significant average linear (Pearson) or monotonic (Spearman) dependence on asset returns. Bitcoin and SPBOND exhibit weak negative Pearson correlation with GPRD, whereas gold exhibits weak positive correlation; but none of these results are significant at 5% level. Likewise, the Spearman coefficients are small and not statistically significant, indicating weak monotonic dependence. The results for green and sustainable assets (Carbon efficient, MSCI SRI, Renewable energy, Clean energy, FTSE Good and Green bond) indicate no significant contemporaneous dependence on GPRD. Overall, these results indicate that traditional correlation analysis fails to capture any significant dependence between geopolitical risk and selected assets, thus motivating the AMFDCCA analysis of nonlinear, asymmetric, and scale-dependent cross-correlations.

Table 4 Correlation Analysis

Asset	Pearson Correlation	Pearson_p_value	Spearman_Correlation	Spearman_p_value
BITCOIN	-0.0331	0.0769	-0.0278	0.1366
GOLD	0.0307	0.1007	0.0294	0.116
SPBOND	-0.0316	0.0913	-0.0298	0.1107
ACWI	-0.0041	0.8275	-0.0116	0.5341
CRUDE OIL	0.007	0.7069	-0.0062	0.7421
CARBON EFFICIENT	0.0178	0.3412	0.0068	0.7148
MSCI_SRI	0.0072	0.7	0.0095	0.6097
RENEWABLE	0.003	0.8737	0.0107	0.568
CLEAN ENERGY	0.0049	0.794	0.0243	0.1938
FTSE GOOD	0.0227	0.225	0.0252	0.1783
GREEN BOND	-0.0054	0.7746	-0.0004	0.9847

ARDL analysis

The benchmark ARDL analysis suggests that geopolitical risk does not play a key role in explaining movements in the selected assets in a linear setting. The model specifications generally do not include lagged GPRD terms, indicating that the lagged linear effect of geopolitical risk is weak. This is also evident from the low adjusted R^2 values, ranging from 0.0018 to 0.023, indicating that the variation in assets is only explained to a limited extent by the benchmark models. Bitcoin is the only asset that displays a statistically significant effect of GPRD (5% significance level) with a negative estimated coefficient of -0.003608. On the other hand, the GPRD effects for all other assets are statistically insignificant and very small. These results suggest that there are only moderate linear relationships between geopolitical risk and the chosen assets in the ARDL models. Therefore, the AMF-DCCA analysis is meaningful as it can reveal more detailed nonlinear, asymmetric, and scale-dependent cross-correlations between the variables. The ARDL benchmark provides little evidence of a linear relationship between GPRD and the chosen assets. The response of Bitcoin is significant but not the other assets. This justifies AMF-DCCA, as it can capture complex dependence.

Table 5 ARDL Analysis

Asset	Selected_p	Selected_q	BIC	Adjusted_R2	Cumulative_GPRD_Effect
BITCOIN	4	0	-8207.57	0.023	-0.003608
GOLD	1	0	-18591.9	0.0018	0.000568
SPBOND	1	0	-24527.6	0.0142	-0.000215
ACWI	1	0	-18901.4	0.0035	-0.000059
CRUDE OIL	4	0	-12060.1	0.0144	0.000388
CARBON EFFICIENT	2	0	-18742	0.005	0.000334
MSCI_SRI	2	0	-18377.3	0.0074	0.000228
RENEWABLE	1	0	-14751.5	0.0068	0.000112
CLEAN ENERGY	1	0	-16003.4	0.0138	0.000133
FTSE GOOD	1	0	-18166.5	0.002	0.000459
GREEN BOND	1	0	-23835.3	0.0113	-0.000033

5. Conclusion

This paper investigates the asymmetric multifractal detrended cross-correlation patterns between green investment funds, conventional financial markets, Bitcoin, and geopolitical risk (GPR), using daily data from October 14, 2013, to September 18, 2025. The results reveal that geopolitical risk has a heterogeneous and scale-dependent association with the selected assets. Gold, Renewable Energy, Bitcoin, Clean Energy, and Green Bonds show the highest scale dependent multifractal behavior, meaning they have the greatest differences between small and large fluctuation dynamics. Conversely, the multifractal responses of MSCI_SRI, FTSE Good, and Carbon Efficient are weaker, indicating relatively more stable dependence structures in the face of geopolitical risk. The multifractality strength of the asset pairs is moderate for ACWI, SPBOND and Crude Oil, indicating that the relationship between the asset pair and geopolitical risk is less strong than for the strongest asset pairs. The results further suggest that geopolitical risk does not affect green investment assets consistently. The response to geopolitical risk is more stable for the broader ESG-based indices (such as MSCI SRI, FTSE Good, and Carbon Efficient) than for the Renewable Energy, Clean Energy, and Green Bonds indices, with some indices exhibiting more pronounced reactions at lower levels of volatility. This distinction is important because it suggests that the geopolitical risk exposure of green investments depends on the specific type of green asset. Within the set of green assets, those more closely tied to renewable energy and the energy transition appear more sensitive to fluctuations, while the larger ESG indices perform more like diversified equity portfolios.

The robustness checks further support the use of AMFDCCA and Pearson Spearman correlations show weak and statistically insignificant contemporaneous relationships between GPR and the selected assets. Similarly, the ARDL benchmark provides limited evidence of linear dependence, with low explanatory power across most models. These results suggest that conventional linear methods may not fully capture the complex dependence between geopolitical risk and financial assets. By contrast, the AMFDCCA results reveal scale-dependent persistence, asymmetric behavior, and heterogeneous responses across asset classes. In terms of the hypothesis proposed earlier, overall, H1 and H2 are partially accepted, while H3 is accepted. The study confirms that geopolitical risk is associated with non-linear, asymmetric, and multifractal dependence between green and traditional financial assets. However, such dependencies differ among assets, GPR trends, and fluctuation scales.

The study contributes to the literature in three main ways: First, it extends geopolitical risk research by examining green investment funds together with conventional financial assets, commodities, and Bitcoin. Second, it shows that green assets respond differently to geopolitical risk, rather than behaving as one uniform asset class. Third, it demonstrates that nonlinear and scale-dependent methods yield richer information than traditional linear approaches for analyzing the transmission of geopolitical risk in sustainable finance markets.

The findings have practical implications for investors and portfolio managers. From an investment perspective, the results suggest that green assets cannot be classified simply as safe-haven or high-risk assets during geopolitical risk. Renewable energy, Clean energy, and Green bonds show notable scale-dependent responses to geopolitical risk, meaning their behavior may change across short- and long-term horizons. Although these assets remain relevant for sustainable investment strategies, their risk exposure may vary considerably across different market conditions. Portfolio managers should evaluate green assets individually before using them for hedging or diversification. Broader ESG indices such as MSCI SRI, FTSE Good, and Carbon Efficient may provide relatively more stable exposure to geopolitical risk compared with specialized green assets. Gold remains relevant for portfolio risk management, although its safe-haven effectiveness appears to vary across different scales of volatility.

Bitcoin requires separate treatment because its dependence on geopolitical risk is unstable and more consistent with that of a speculative digital asset than of conventional financial indices. Accordingly, these results suggest that risk-management frameworks should account for geopolitical risk and nonlinear cross-asset dependence, rather than depending solely on average correlation measures. In periods of heightened geopolitical uncertainty, portfolios containing Renewable energy, Clean energy, and Green bonds should be stress-tested to evaluate their contribution to broader risk exposure.

The findings also indicate that sustainable finance markets remain not entirely immune to geopolitical turmoil. Policymakers and regulators should strengthen risk-disclosure requirements for green investments, including Clean Energy, Renewable Energy, and Green Bonds. These disclosures need to account for geopolitical risk, energy market instability, policy uncertainty, and other macro-financial shocks. To further strengthen and deepen the green finance market, regulatory authorities should consider providing stronger support, such as encouraging its growth. In addition, the market can also be strengthened and deepened by encouraging the growth of the green finance market, which can help stabilize the green finance market during geopolitical turmoil and enhance the confidence of investors. Such sustainable finance policies should be linked to energy-security and financial-stability policies, as disruption in traditional energy and commodity markets may affect green financial instruments. The findings highlight the importance of stable climate policy frameworks for policymakers. Clear renewable energy policies, reliable green bond standards, and transparent ESG reporting can reduce investor uncertainty as geopolitical risks intensify.

There are several limitations to this study. First, the analysis is limited to a selected group of green investment funds, conventional financial indices, commodity indices, and Bitcoin. The conclusions are thus not necessarily generalizable to all emerging markets, regions, or types of green finance instruments. Future research may extend the analysis by including regional green bond indices, ESG exchange-traded funds, climate-transition funds, and emerging-market green finance assets. Second, AMF-DCCA can be applied to identify nonlinear, asymmetric, and scale-dependent cross-correlations, but it cannot conclude direct causality. The approach detects dependencies between geopolitical risk and the chosen assets, but it cannot determine whether geopolitical risk drives market movements. Further research can be carried out by applying other nonlinear causality techniques, such as multifractal analysis, nonlinear Granger causality, quantile causality, and time-varying connectedness models. Third, the Pearson correlation, Spearman correlation, and ARDL benchmarks are employed as robustness checks, although the methods primarily describe linear or average dependence. These additional robustness analyses can be performed in the future, for instance, using rolling-window AMF-DCCA, wavelet coherence, quantile coherence, or subsample analyses around geopolitical events. Fourth, the study adopts a wide geopolitical risk index. This index does not specifically measure geopolitical uncertainty and may not be a good indicator of the impact of certain events, such as regional wars, sanctions, trade conflicts, or energy-security crises. Further research can then investigate whether there are differences in how various geopolitical risk events affect those green assets, in particular by using regional or event-specific geopolitical risk measures.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding

The research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Data Fabrication/Falsification Statement

The author(s) declared that no data has been fabricated, falsified, or manipulated in this study.

Participant Consent

This study is based on secondary data obtained from publicly available sources and did not involve any human participants. Therefore, no participant consent was required, and all data was used in accordance with ethical standards for secondary data research.

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