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Optimising the Investor's Portfolio through Modern Portfolio Theory: Empirical Evidence from Pakistan

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Abstract

Diversification is considered a way to lower investment risk by using a variety of investment avenues. The objective of this study is to compare the portfolio performance based on mean-variance optimisation with a naively diversified portfolio for textile spinning stocks in the Pakistan country. The data period of study spans from January 2015 to April 2022. The result shows that portfolios based on naïve diversification strategies outperformed the mean-variance portfolio optimisation in terms of risk and return for the textile sector in Pakistan. Of all the 9 variance-covariance estimators, the portfolio of sample and single index covariance technique is optimal for estimating variance-covariance matrices based on low RMSE (root mean square error) in Pakistan. This study extends the debate on mean-variance portfolio theory and naïve diversification strategy from the developed economy to the emerging equity market of Pakistan. This research contributes by providing the framework to the potential investors regarding their investments in the textile sector in Pakistan.

Introduction

Markowitz was the first to propose the concept of mean-variance optimisation (1952). Despite the fact that research like Jagannathan and Ma (2003) and Chan, Karceski, and Lakonishok (1999) support the use of the standard mean-variance paradigm for optimal portfolio management, it has been called into question on several fronts. Michaud (1989) refers to the concept as a "enigma," although Disatnik and Benninga (2007) argue that it produces dubious results. There are two approaches to dealing with the issues that standard mean-variance optimisation presents. The theoretical technique focuses on the assumptions and theoretical features of the mean-variance framework. In contrast, the implementation technique looks into how investors might estimate the expected return vector and covariance matrix of asset

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classes in order to properly apply the framework.

Beginning in the early 1960s, financial analysts began to assess risk more rigorously (Husnain, Hassan, & Lamarque, 2016 a, b). However, there is currently no reliable mechanism for measuring risk. Professor Harry Markowitz's (1952) corpus of work lays out the core theories underlying portfolio theory. Despite the fact that Markowitz's (1952) work is statistical in nature, the notion that drives his asset allocation approach may be described using the saying "don't put all your eggs in one basket." According to Markowitz, the variance in rate of return is a sufficient measure of portfolio risk if certain reasonable assumptions are made. In addition, he creates a computational formula. The portfolio risk formula highlights the importance of diversification in reducing overall portfolio risk. Many essential assumptions underpin Markowitz's (1952) model of investor behaviour. For example, an investor optimises one-period expected utility while the utility curve reduces wealth's marginal value. Variability in expected returns is a statistic for estimating portfolio risk. Furthermore, it is assumed that investors make their decisions only based on the expected return and risk of an investment. Investors would prefer higher returns for the same amount of risk than lower returns for the same level of risk. Investors, on the other hand, would choose lower risk for the same level of return than higher rates of return for the same level of risk. According to Markowitz (1952), the expected return of a portfolio is defined as the weighted average expected return of all of the portfolio's constituent assets. The risk of a portfolio (as measured by standard deviation) is influenced not just by the risk of individual assets, but also by the covariance between the returns of all pairs contained in that portfolio. The degree to which the return on one asset is connected with the return on other assets is seen as the single most important aspect by investors.

Investors may encounter various situations during the asset allocation (Ahmad, 2020; Umar et al., 2021; Wang, et al., 2021). An investor can use the mean-variance (MV) optimisation principle and minimum variance portfolios (GMVP) throughout the asset allocation process. Furthermore, the investor may sometimes invest equally in all accessible capital investments resulting in naively diversified portfolios (NDs). As a result, investors must make decisions in numerous areas simultaneously, determining each class's optimal weight (Ashraf et al, 2020).

The main objective of this study is to compare the portfolio risk and return based on a naively diversified portfolio with mean-variance portfolio (MVP) optimisation in the textile sector in Pakistan and to find the optimal ways to estimate variance-covariance matrices in Pakistan. Because the minimum variance portfolio is unaffected by choice of return vector, we utilise its risk profile to compare different covariance matrices. We also compute the risk-return characteristics of the mean-variance portfolio, such as the number of positive and negative weights, the maximum and minimum weight values, and the excess sharp ratio (ESR) and the Herfindahl index. This study extends the debate on mean-variance portfolio theory and naive diversification strategy from the developed economy to the emerging equity market of Pakistan. This study contributes by guiding the potential investor investing in the textile sector in Pakistan. Modern portfolio theory (MPT) also helps decide how investors invest in different stocks or firms (Ahmad, 2020; Umar et al., 2021; Wang, et al., 2021).

Asset allocation is an important component of the portfolio management process. As a result, investors must select a suitable asset class combination. This research assists investors in identifying and investigating all accessible investment possibilities in Pakistan. This deployment of resources across asset classes eventually leads to portfolio diversification for investors. The most important question for investors when it comes to diversification is how many stocks make up a diverse portfolio. Investors do this by comparing the marginal benefit to the marginal cost and determining the optimum asset class mix. Investors can distribute their whole investment proportionally or disproportionately to other asset groups. Investors have the choice of distributing their whole investment based on naïve diversity or finding a certain amount of diversification. The study also provides the best methods for estimating inputs for portfolio optimization, such as the estimate of the return vector and the variance-covariance matrix, which consistently beat rival methods in Pakistan. It assists investors in allocating their funds to various hazardous investment options.

In the next section, "Section 2," we will detail the dataset, the study methods, and our criteria for comparison. The empirical findings of the study are presented in Section 3, which is then followed by a discussion of the results in Section 4, and finally, Section 5 brings the work to a close.

Dataset and Research Methodology

This research offers investors in the textile industry in Pakistan a methodology for asset allocation based on their specific needs. The data set is made up of time series information that is connected to the various asset classes. In addition, the information that pertains to the firms that are listed on the Pakistan stock market (PSX) was gathered from the official website of the PSX in Pakistan. The data have been taken for the largest 15 listed companies based on the market capitalisation of PSX in the textile spinning sector in Pakistan. The data have been taken for 7 years from 1 January 2015 to 7 April 2022.

This paper gives investors a detailed method for allocating assets among several securities. The equation is used to compute continuously compounding returns for each class.

$$\mathrm{Rt} = \ln\left(\mathrm{Pt} / \mathrm{Pt} - 1\right) \ldots\ldots\ldots (1)$$

Rt = return of the continuously compounding
Pt = Price at period "t"
P(t-1) = Price at time period “t-1”
ln = Log of the given value

Historical Average estimation

The following method is used to determine the simple arithmetic mean across the investigated period (M) of each asset class to estimate future returns E(Ri), Ri represents the asset class's historical return 'i'.

$$\mathrm{E(Ri)} = 1/\mathrm{M} \sum_{t=1}^{M} \mathrm{Ri, t} \ldots\ldots\ldots (2)$$

Wherever:

E (Ri) = Expected/Estimated return
Ri = Asset class historical return "t"
M = number of time periods

Variance-covariance matrix (VCM)

The asset class expected return and covariance matrices are the two most important parameters that investors must provide for the portfolio optimisation process that determines the proportion invested in each asset class. According to DeMiguel, Garlappi and Uppal (2009), calculating the covariance matrix is essential for portfolio covariance optimisation.

The sample variance-covariance matrix (VCM) may be calculated using the following formula.

$$\mathrm{VCM} = (1/(\mathrm{K}-1))\mathrm{R}(\mathrm{I}-1/\mathrm{K}\,11')\mathrm{R}' \;\ldots.. \;(3)$$

The sample variance-covariance matrix has the advantage of being the most likely under the normality assumption—the likelihood of overfitting the data increases as the sample size diminishes. So, compared to out of sample, its performance for the sample is better. The variance may be calculated as follow.

$$\mathrm{Mij} = (\mathrm{K}-1)^{-1}\sum_{t=1}^{K}[\mathrm{R}_{i,t}-\overline{\mathrm{R}}_{t}][\mathrm{R}_{j,t}-\overline{\mathrm{R}}_{j}],\;\; i,j = 1,2,3,\ldots.,\mathrm{N} \;\ldots\ldots (4)$$

Where:

$$\overline{\mathrm{R}}_{t} = \frac{1}{K}\sum_{t=1}^{k}\mathrm{R}_{i,t} \quad, i = 1,2,3,\ldots,\mathrm{N} \;\ldots.. (5)$$

This study considers the following 9 variance-covariance estimators as an input to portfolio optimisation.

Table 1: List of variance-covariance estimators

Covariance Estimators	Abbrevations
Diagonal Covariance	VC-1
Sample Covariance	VC-2
Constant Correlation Covariance	VC-3
Single Index Covariance	VC-4
Sample and Diagonal Covariance	VC-5
Sample and Single Index Covariance	VC-6
Sample and Constant Correlation Covariance	VC-7
Sample, Single Index and Constant Correlation Covariance	VC-8
Sample, Single Index, Constant Correlation and Diagonal	VC-9

Traditional mean-variance framework

Modern portfolio theory is the result of Markowitz (1952, 1959) demonstration of a statistical method for choosing a portfolio. Markowitz assumes that investors are risk averse i.e., they always desire minimal risk for a given return level. The mean-variance framework is described in detail as follows.

Risk and return of the portfolio

If there are "N" different asset classes, and "Wi" represents the percentage of total capital that is allocated to "I" of those asset classes, then the expected return of a portfolio can be calculated as follows:

$$\mathrm{E(Rp)} = w_1\mathrm{E(R)}_i + w_2\mathrm{E(R)}_2 + \cdots + w_N\mathrm{E(R)}_N = \sum_{i=1}^{N} w_i\,\mathrm{E(R)}_i \;\ldots.. (6)$$

where:

E (Rp) = Portfolio of Expected Return
Wi = the weight' **i’**
E (R)i = Asset Class Expected Return ‘i’
The portfolio’s expected return can be expressed as follows in matrix notation:

$$E(Rp) = ∑_(i = 1)^N * wi 〖E(R)i = W^T E(R) \dots (7)$$

Where:

$$W = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_N \end{pmatrix} \ \& \ E(R) = \begin{pmatrix} E(R_1) \\ E(R_2) \\ E(R_3) \\ \vdots \\ E(R_N) \end{pmatrix} \dots (8)$$

The weighted average return of the portfolio’s various asset classes is known as a portfolio return. However, the logarithm property states that a sum’s logarithm is not equal to the sum of its logarithms. As a result, the weighted average of asset class returns is almost equivalent to the continuously compounded portfolio return. The distribution of returns for the upcoming month is predicated on data from previous sample periods. If there are “N” asset classes and “wi” represents the percentage of money invested in asset class I, then the portfolio’s variance may be calculated as follows:

Portfolio variance = $w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov1,2$

$$Var\ (R_P) = \sum_{i=1}^{N}(W_i^2)\ Var\ (R_i) + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} W_iW_J Cov\ (R_i\,, R_j) \ \dots. (9)$$

Where:
Var (Rp) = Standard deviation of portfolio while Cov (Ri, Rj) denotes the covariance. Also,

$$Var(R_p) = \sum_{i=1}^{N}\sum_{j=i+1}^{N} W_i\, W_j \sigma_{ij} \dots (10)$$

Where:

$$Var(R_p) = [W_1 \quad W_2 \quad W_3 \dots . W_N] \begin{bmatrix} \sigma_{12} & \sigma_{12} & \sigma_{13} & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \dots & \sigma_{2N} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{3N} \end{bmatrix} [\begin{matrix} W_1 \\ W_2 \\ W\ _3 \\ W_N \end{matrix}] \dots (11)$$

$$Var\ (R_p) = W^T \times SW \dots. (12)$$

WT is the transpose of the weights matrix, S is the variance-covariance matrix, and W is the weights matrix. The term "Correlation" refers to the relationship between the rates of return on asset classes I and "j" in this calculation.

$$\text{Correlation Coffiecient} = \rho_{i,j} = \frac{\sum_{t=1}^{N}[R_{i,t} - E(R_i)][R_{j,t} - E(R_j)]}{\sqrt{\sum_{t=1}^{N}[R_{i,t} - E(R_i)]^2}\ \sum_{t=1}^{N}[R_{j,t} - E(R_j)]^2} \dots. (13)$$

The following formula can be used to explain the connection between covariance and correlation coefficient.

$$Corr = \rho_{i,j} = \frac{Cov\ (r_i, r_j\ )}{\sigma_i, \sigma_j} \dots.(14)$$

Evaluation Dimensions

According to Liu and Lin (2010) and Jagannathan and Ma (2003), we compare covariance matrices using two different types of evaluation standards. The first is the root mean square error (RMSE), while the second is the sharp ratio. First, we compute the covariance matrices based on the first subsample window, and then we examine the pair-wise accuracy of the estimators using the root-mean-squared error. The ex-post precision of the covariance matrix can be determined using the second subsample window. To do so, it is required to evaluate the differences in the covariance estimators generated by the two subsample windows. Therefore, this study examines nine covariance estimators based on these two standards to align the results with the existing literature. To compare the pair wise estimate accuracy of covariance's, the following method is utilised to calculate RMSE in line with Liu and Lin (2010).

$$\mathrm{RMSE} = \sqrt{\frac{N(N-1)}{2}} \sum_{i=1}^{N} \sum_{j=1,\ i\neq 1}^{N} (\hat{\sigma}_{ij} - \sigma_{ij})^2 \ldots. (15)$$

In the first place, this ratio illustrates the proportion of returns to risk, which can be of utmost importance for portfolio investments (Sharp, 1963). It was determined by applying the formula that is presented below.

$$\mathrm{S\ Ratio} = (R_p - r_f) \div \sigma_p \ldots \ldots (16)$$

A lower RMSE is preferable to a greater RMSE. Because MVP is unaffected by choice of return vector, we utilise its risk profile to compare different covariance matrices. We also compute the risk-return characteristics of the mean-variance portfolio, such as the number of positive and negative weights, the maximum and minimum weight values, and the excess sharp ratio (ESR). The Herfindahl index is also computed in this study (HI).

The following figures show the conceptual framework of this study.

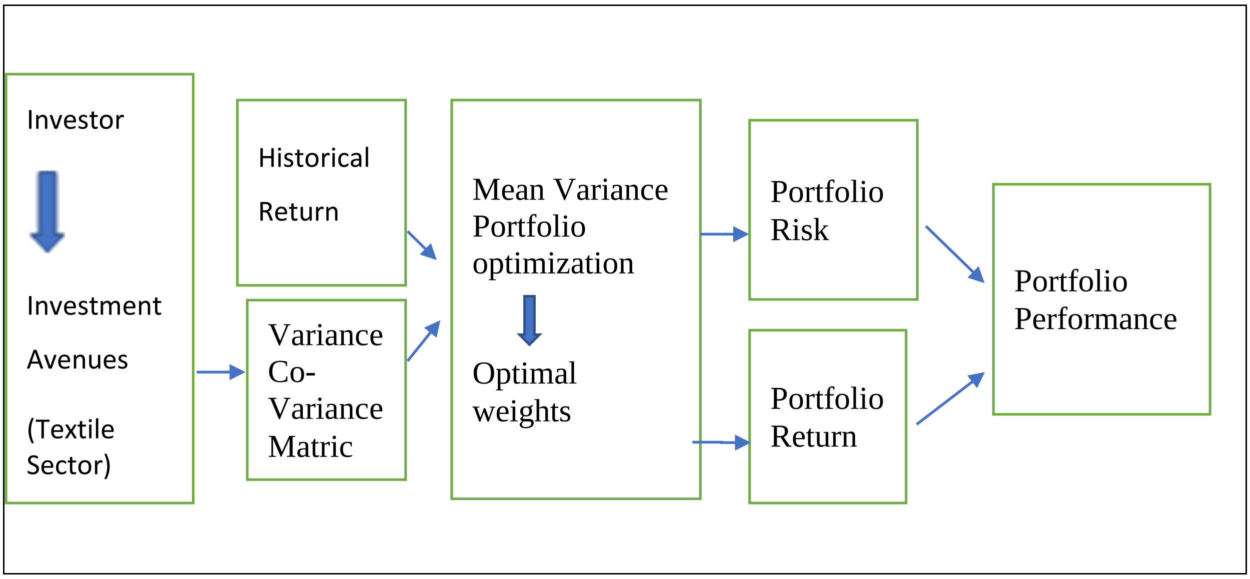

Figure: 1 Mean-Variance Portfolio Optimization

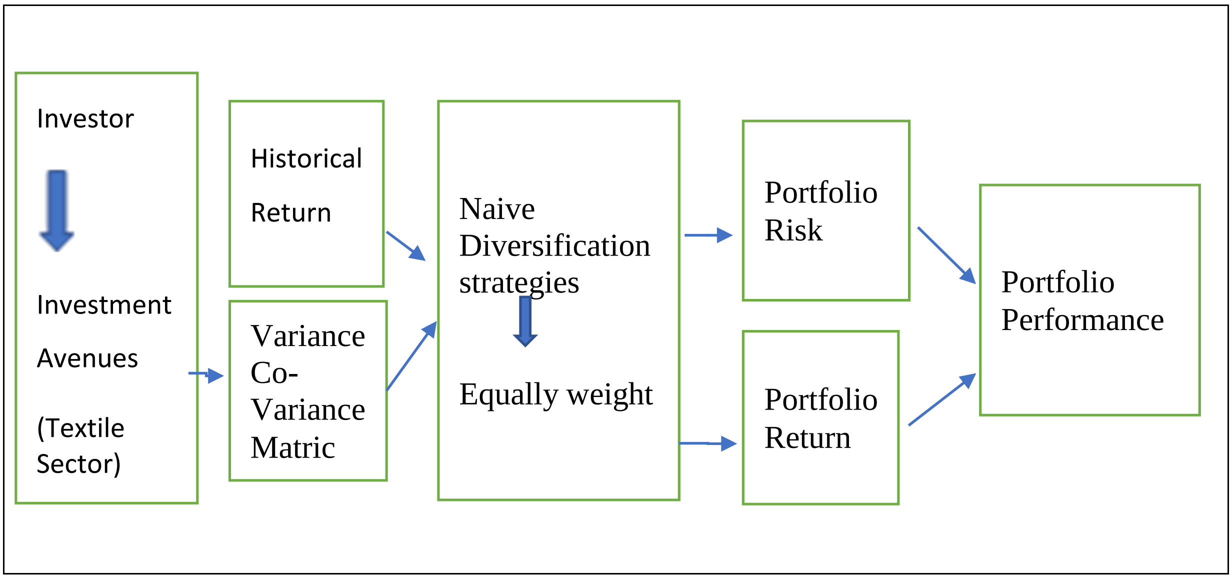

Figure: 2 Naive diversification strategy (NDs) for weight Computation

Empirical Findings

This section presents the study's empirical findings. It includes results on asset allocation frameworks in the textile sector in Pakistan.

Empirical Finding of Textile Spinning Sector

This study examines 9 covariance matrices based on descriptive statistics from the spinning sector, and Table 2 displays the findings. A covariance matrix indicates the mean, standard deviation, lowest and maximum value of the entire 15-textile spinning sector. It demonstrates that if an investor takes a high risk, they can expect a huge return, but they can also expect a significant loss on that increased risk. The positive value of the mean (ASGF = 0.0713, SD = 0.3401) shows that investors get a good investment return. The negative value means (HTML of mean = -0.0160, SD = 1.3426) shows that investors lose from that investment.

Table 2: Descriptive Statistic Spinning sector

	Mean	SD	Minimum	Maximum
ASGF	0.0713	0.3401	-0.9997	1.287
BFL	0.0524	0.4939	-1.8695	2.9474
CSML	-0.0399	1.0885	-.5884	2.5884
CTML	0.0289	0.4568	-0.9994	1.9972
D.SIL	0.0042	0.7275	-2.8876	1.9927
DSML	0.0031	0.5000	-1.8163	2.8163
DMTM	-0.0035	0.5411	-2.6190	2.2565
DTM	0.0484	0.7388	-2.7247	2.9176
GTM	0.0222	0.9281	-2.4558	2.3243
HTML	-0.0160	1.3426	-2.6600	1.6515
KSML	0.0026	0.9545	-1.4123	2.4122
NCML	-0.0073	0.3126	-2.8436	1.1894
RTML	-0.0122	0.5451	-2.9555	2.8367
STML	0.0025	0.6073	-1.7393	1.9196
SSML	0.0003	0.4685	-1.5324	2.3979

Table 3 displays the beta value for Pakistan's overall 15-textile spinning sector. The beta is a term that measures a stock's predicted to move in relation to moves in the textile sector. If the beta value is larger than one, it indicates that the stock is more volatile in the textile sector. If the beta value is less than one, the stock in the textile spinning industry is less volatile. If the beta value is larger than one, the return on the spinning sector is more variable than the market return on the textile sector, and it is also an aggressive stock to invest in. If the beta value is less than one, the return on the spinning industry is less variable than the market return of the textile sector and it is also defensive stock.

Table 3: Beta of Sample Stocks

	Beta
ASGF	0.0025
BFL	0.0062
CSML	0.0181
CTML	0.0028
D.SIL	0.0084
DSML	0.0065
DMTML	0.0003
DTM	0.0038
GTML	0.0144
HTML	0.0308
KSML	0.0239
NCML	0.0054
RTML	0.0100
STML	0.0008
SSML	0.0171

Empirical Finding of Minimum Variance Portfolio Weight (GMPV) in Spinning Sector

The result of table 4 shows the weights for minimum variance portfolios in the spinning sector of Pakistan. This study compares the 9 covariance matrices based on 15 textile spinning sectors using the minimal variance portfolio method (GMVP). In this study, all 9 covariances are compared with all 15 textile spinning sectors from various perspectives. The variance-covariance finding for all 15 textile spinning sectors is different from each other.

Table 4: Minimum Variance Portfolio Weight (GMVP) Spinning Sector

VCM	VC-1	VC-2	VC-3	VC-4	VC-5	VC-6	VC-7	VC-8	VC-9
ASGF	0.0749	0.0696	0.0587	0.0746	0.0725	0.0724	0.0796	0.0908	0.0990
BFL	0.0205	0.0177	0.0671	0.0205	0.0190	0.0190	0.0748	0.0759	0.0755
CSML	0.0017	0.0015	0.0687	0.0017	0.0016	0.0016	0.0289	0.0203	0.0162
CTML	0.0175	0.0250	0.0664	0.0177	0.0211	0.0212	0.0754	0.0768	0.0762
D.SIL	0.0042	0.0051	0.0626	0.0043	0.0045	0.0046	0.0448	0.0369	0.0321
DSML	0.0132	0.0034	0.0525	0.0132	0.0084	0.0084	0.0569	0.0565	0.0551
DMTML	0.0167	0.0151	0.0682	0.0165	0.0158	0.0157	0.0738	0.0729	0.0713
DTM	0.0083	0.0027	0.0521	0.0083	0.0054	0.0054	0.0481	0.0448	0.0419
GTML	0.0106	0.0118	0.0703	0.0104	0.0111	0.0111	0.0703	0.0651	0.0606
HTML	0.0012	0.0009	0.0647	0.0011	0.0010	0.0010	0.0192	0.0134	0.0108
KSML	0.0054	0.0061	0.0736	0.0054	0.0058	0.0058	0.0605	0.0501	0.0434
NCML	0.1576	0.1384	0.0561	0.1580	0.1490	0.1491	0.0802	0.0935	0.1039
RTML	0.6115	0.6426	0.0805	0.6113	0.6262	0.6262	0.1015	0.1143	0.1255
STML	0.0186	0.0183	0.0819	0.0187	0.0183	0.0184	0.0903	0.0880	0.0849
SSML	0.0378	0.0414	0.0761	0.0378	0.0396	0.0396	0.0949	0.1003	0.1031
Sum of weight	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Empirical Finding of Efficient Weight Markowitz in Spinning Sector

The result of table 5 shows the weights for minimum variance portfolios in the spinning sector of Pakistan. This study also compares the 9 covariance matrices based on 15 textile spinning sectors using the efficient weight method based on Markowitz theory. In this study, all 9 covariances are compared to the entire 15-textile spinning sector from various perspectives. The result variance-covariance of all 15 textile spinning sectors is different.

Table 5 Markowitz mean-variance (Efficient weight) Spinning Sector

VCM	VC-1	VC-2	VC-3	VC-4	VC-5	VC-6	VC-7	VC-8	VC-9
ASGF	0.4892	0.5075	0.3842	0.4898	0.4995	0.4998	0.4331	0.4588	0.4792
BFL	0.0542	0.0466	0.1417	0.0542	0.0504	0.0504	0.1409	0.1365	0.1323
CSML	0.0011	-0.009	0.0299	0.0011	0.0001	0.0001	0.0104	0.0067	0.0052
CTML	0.0569	0.0608	0.1886	0.0575	0.0587	0.0589	0.1841	0.1743	0.1659
D.SIL	0.0023	0.0025	0.0145	0.0025	0.0021	0.0022	0.0074	0.0078	0.0076
DSML	-0.018	-0.037	-0.125	-0.018	-0.027	-0.027	-0.115	-0.101	-0.091
DMTML	0.0248	0.0273	0.0876	0.0245	0.0261	0.0259	0.0845	0.0801	0.0763
DTM	0.0164	0.0142	0.1081	0.0164	0.0151	0.0151	0.0877	0.0750	0.0667
GTML	0.0140	0.0001	0.0494	0.0138	0.0069	0.0068	0.0405	0.0361	0.0333
HTML	0.0001	0.0003	0.0038	0.0001	0.0001	0.0001	0.0016	0.0007	0.0005
KSML	-0.002	-0.002	-0.022	-0.002	-0.002	-0.002	-0.015	-0.012	-0.0101
NCML	-0.031	-0.068	-0.041	-0.030	-0.049	-0.049	-0.042	-0.041	-0.0403
RTML	0.3549	0.4121	0.0610	0.3530	0.3823	0.3814	0.0649	0.0663	0.0681
STML	0.0339	0.0341	0.1168	0.0342	0.0344	0.0345	0.1141	0.1070	0.1012
SSML	0.0032	0.0030	0.0026	0.0033	0.0032	0.0033	0.0041	0.0047	0.0051
Sum	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Empirical finding of characteristics of weight Under Minimum Variance Portfolio weight (GMPV) in the Spinning Sector

Table 6 shows the results of weight characteristics under minimum variance portfolios. It contains portfolio return and risk, excess sharp ratio, the value of the Herfindahl index, number of positive and negative weights under 9 alternative covariance inputs to positive variance portfolios. The VC-6 clearly outperforms the VC-7, since VC-6 has a lower standard deviation with a greater portfolio return.

Table 6: Portfolio Characteristic of GMVP (Spinning Sector)

VCM	Portfolio Excess Return	Portfolio Excess SD	Portfolio Excess Sharp Ratio (ESR)	Herfindahl Index (HI)	No of Positive Weights	No of Negative Weights
VC1	0.0150	0.0604	0.2476	0.4076	15.000	0.0000
VC2	0.0151	0.0610	-0.1034	0.0404	15.000	0.0000
VC3	0.0187	0.1959	0.0955	0.6679	15.000	0.0000
VC4	0.0149	0.0604	0.2472	0.4075	15.000	0.0000
VC5	0.0150	0.0606	0.2478	0.4229	15.000	0.0000
VC6	0.0190	0.0601	0.2479	0.7678	15.000	0.0000
VC7	-0.0213	0.1306	-0.1630	0.0744	15.000	0.0000
VC8	-0.0222	0.1187	-0.1869	0.0788	15.000	0.0000
VC9	0.0127	0.1120	0.2027	0.0826	15.000	0.0000

Characteristics of Efficient Weights Markowitz in Spinning Sector

Table 7 shows the results of the features of efficient weights in Markowitz mean variance portfolios. It contains portfolio return and risk, excess sharp ratio, Herfindahl index (HI) value, number of positive and negative weights under 9 covariance alternative inputs to Positive variance portfolios. The equally weight covariance estimator (vc-6) clearly outperforms the covariance estimators (vc-7) since vc-6 has a lower standard deviation with a greater portfolio return. The vc-8 has the lowest excess sharp ratio. The single index base covariance estimator has a higher excess sharp ratio than the constant correlation base covariance estimator. In addition, sample base covariance estimators (vc-2) are outperformed by equally weighted covariance estimators (vc-6 and vc-7), based on the excess sharp ratio. A further point of interest is that the sample covariance (vc-2) has the highest Herfindahl index value. Other explored covariance estimators have a much lower short position than sample base covariance estimators.

Table 7:Portfolio Characteristic of Efficient Wight by Markowitz (Spinning Sector)

VCM	Portfolio Excess Return	Portfolio Excess SD	Portfolio ESR	HI	No of Positive Weights	No of Negative Weights
VC1	0.0586	0.1195	0.4899	0.3751	12.0000	3.0000
VC2	0.0609	0.1253	0.4863	0.7414	11.0000	4.0000
VC3	0.0654	0.1878	0.3483	0.2616	12.0000	3.0000
VC4	-0.0586	0.1197	0.4899	0.3744	12.0000	3.0000
VC5	0.0598	0.1223	0.4890	0.4071	12.0000	3.0000
VC6	0.0899	0.0224	0.6890	0.0451	12.0000	3.0000
VC7	0.0686	0.1788	0.3835	0.2907	12.0000	3.0000
VC8	-0.0694	0.1721	-0.4031	0.3010	12.0000	3.0000
VC9	0.0700	0.1679	0.4170	0.3112	12.0000	3.0000

Characteristics of Naïve Diversification in the Spinning Sector

Table 8 shows the risk and return of naive diversification portfolios with 15 asset classes investors. It also displays the Herfindahl index (HI), the excess sharp ratio (ESR), the number of positive weights, the number of asset classes with short sales, and the range of weights under the naive diversification equally conditions by varying the variance-covariance estimators (Ledoit and Wolf (2020, 2021). In Pakistan, the constant correlation base covariance matrix and single index basis covariance matrix have similar ESR values to other covariance estimators.

Table 8: Characteristics of Naïve Diversification

VCM	Portfolio Excess Return	Portfolio Excess SD	Portfolio ESR	HI	No of Positive Weights	No of Negative Weights
VC1	0.0167	0.1283	0.13012	0.0667	15.000	0.000
VC2	0.0131	0.1496	0.08751	0.0667	15.000	0.000
VC3	0.0136	0.3237	0.04201	0.0667	15.000	0.000
VC4	0.0190	0.1295	0.14671	0.0667	15.000	0.000
VC5	0.0145	0.0656	0.2211	0.0667	15.000	0.000
VC6	0.2190	0.1399	1.56473	0.0667	15.000	0.000
VC7	0.0154	0.2522	0.06106	0.0667	15.000	0.000
VC8	0.0128	0.2191	0.05843	0.0667	15.000	0.000
VC9	0.0195	0.2003	0.09736	0.0667	15.000	0.000

Empirical Evidence of Root Mean Square in Spinning Sector

The root means square error methodologies support measuring and estimating data correction. The standard deviation of the prediction error is the root mean square error. Lower RMSE values suggest a better outcome. The diagonal (vc-1) shows that (RMSE = 0, Average = 0, SD = 0.2025).

Table 9: Root mean square error (RMSE) Spinning Sector

	RMSE
VC-1	1.0011
VC-2	3.4403
VC-3	8.9407
VC-4	0.3839
VC-5	1.7352
VC-6	0.0202
VC-7	5.8838
VC-8	3.9285
VC-9	2.9464

Conclusion and Recommendation

The process by which an investor allocates their capital among different asset classes is called asset allocation. Markowitz (1952) developed an optimal formula for asset allocation. This research article is concerned with estimating a covariance matrix as an input to asset allocation and using mean-variance criteria for tactical asset allocation decisions in Pakistan's equities market. It has two key goals in particular. First, it examines 9 various methods for estimating the covariance matrix in Pakistan's equities market. It compares covariance matrices using two different sorts of evaluation criteria. The first is the root mean square error, and the computation of portfolios with the lowest variance. Second, it compares the optimal weights by mean-variance framework with weights under non-theory base diversification based on the risk-return characteristics of the mean-variance portfolio, number of positive and negative weights, excess sharp ratio, and Herfindahl index.

The result shows that the portfolio risk based on naïve diversification strategies are outperformed the mean-variance portfolio (MVP) optimisation in the textile sector in Pakistan. The result of the portfolio return based on naïve diversification strategies are outperformed the mean-variance portfolio (MVP) optimisation in the textile sector in Pakistan. Of all the 9 variance-covariance matrix techniques, the sample and single index-based covariance estimation (VC- 6) outperformed, and the VC-6 is optimal for estimating variance-covariance matrices based on low RMSE in Pakistan.

The study recommended that sample and single index-based covariance estimation is a good method that outperformed with respect to other variance-covariance metrics and equal weight techniques that outperformed with respect to another mean-variance and minimum variance portfolio based on excess sharp ratio. According to the study, investors in emerging nations such as Pakistan should use sample and single index models for estimating the variance-covariance matrix as an input to portfolio optimisation. It also suggests that an equally weighted portfolio is more appropriate than the mean-variance portfolio optimisation in the textile sector in Pakistan. The study also suggests that investment managers and academics view naive diversification as the first obvious benchmark in contrast to other asset allocation techniques. For future researchers, it is suggested that these portfolio estimation strategies can be extended to other industries in Pakistan and to different emerging and developed economies. Furthermore, this study is limited to only the textile sector of Pakistan. Also it only employed the daily data of the listed stocks in textile sector of Pakistan. When developing optimum portfolios, further study might consider the influence of higher-order moments. We also urge that investors construct stronger comparison criteria for the variance-covariance matrix, because the RMSE simply takes into account individual changes in each member of the matrix, but a better gauge would take into account the matrix's overall structure. Furthermore, in terms of asset allocation, the MVP is merely one portfolio on the efficient frontier. This indicates that additional criteria are required to assess different covariance estimators in order to achieve more satisfying findings.

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